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A STUDY OF THE PECULIAR CEPHEID HR7308

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A STUDY OF THE PECULIAR CEPHEID HR7308

Göran Henriksson

Key-words:Variable amplitude,Cepheid,photoelectric photometry, radial velocity,Fourier analysis,period variation,stellar evolution,radius determination,mass determination,nonradial oscillations.

INTRODUCTION

The author has performed a survey for new small amplitude Cepheids using the 40cm photoelectric reflector at the Kvistaberg observatory belonging to the astronomical observatory in Uppsala. The survey started during the spring 1972 and after two years the project was continued by observations of the suspected and certain variable stars. Some preliminary results have been published(Henriksson 1977) for the main survey containing 121 stars. A smaller survey containing only 20 stars selected from a list of MK-classifications Harlan(1969) was started before the main project and for these stars six observations were made relative to comparison stars to test the stability of the light. All the program stars are expected to be situated within the instability strip in the HR-diagram and the stars found to be constant were not uninteresting in this case. The most interesting of the variable stars found in this smaller survey is HD180583 = HR7308 discovered in 1972. It was classified as a small amplitude Cepheid with a very short period 1.5 days. The first problem with this star was the period itself because the star can only be observed during 5 hours per night from Kvistaberg and after three days the same part of the light-curve is observed again. This means that it is only possible to observe three 5 hour intervals of the 36 hour period. A combination of the observations during 1972 and 1973 made it possible to determine a period of 1.491 days and an amplitude of 0.11 magnitudes. The observations during 1974 were,however,an disapppointment from this point of view because they fell all below the predictions. The observations were unfortunately made only in minimum and with daynumbers moduli 3 and therefore it was impossible to judge whether the star was eclipsed by another star or the amplitude had increased. The observations from 1975 were similar to the observations from 1972-1973,and the period 1.491 days satisfied also the new observations from 1976. An analysis of the observations with respect to time-shifts caused by a possible motion in a binary system suggested an orbital period of 3-7years. It was therefore very interesting to observe the star during 1977 and three observations were made in April and May but they showed only a small increase in the amplitude. In August 1977 the increase was definite and the eclipse theory could be ruled out. The amplitude was increasing during the rest of 1977 and reached a maximum during March 1978. When it was real-

ized that the amplitude could become as great as 0.32mag it was possible to use good visually estimated magnitudes if they were made during amplitude maximum. In fact 15 observations that can be used have been found in the Harvard and Potsdam photometries from 1880-1897. The oldest photoelectric observations in the UBV-system are from 1962 by A.N. Argue(Argue 1963) and he has kindly sent me the three individual observations. They are extremely important for the precise determination of the period for the amplitude variation because they are made during an amplitude maximum. It is also possible to get information about the period for the amplitude variation from published radial velocities. In the beginning of October 1979 the author was fortunate to have five nights at the 152cm coude' telescope at l'observatoire de Haute Provence in France and it was possible to get one good spectrum every night at a dispersion of 12Å/mm. The radial velocity amplitude was found to be 6.4km/s from these observations made during the low-amplitude phase. It was thereafter clear that the observations from 1929 by Albitzky and Shajn(1933) were made during a high-amplitude phase. From this information we can determine the period of the amplitude variation to be 1178.5days=3.2266years. Photometric observations during 1979-1981 have confirmed this period and a new maximum occured as expected in June 1981. The author suggests that the strictly periodic variations in amplitude are caused by a small companion moving in an elliptical orbit around the Cepheid. Finally variations of the pulsation period of the Cepheid has been found. Two periods are required to describe these variations. The first variation has a period that is equal to the period for the amplitude variation and the second is 2.25 times that period. A secular increase in the pulsation period has also been determined and can be described by

PC=0.00000122*(J.D.-2443600)/365.25+1.490795days.

The data have been analysed by a modified Fourier analysis. Preliminary results from the author's investigation of this star have been presented at the I.A.U. Fifth European Regional Meeting in Lie'ge, Belgium 1980.

THE HISTORY OF DISCOVERY

The first known comment about the variability of HR7308 was made by Breger(1969). He classified the variations during 10 observations 1966 and 1967 as "semi-regular". In 1972 the author of this paper started a survey for new low amplitude variable stars and stable stars within the Cepheid instability strip. This observational program should continue the investigations by Fernie and Hube(1971) and a preliminary survey containing 20 stars selected as possible variable stars from their positions in the instability strip was started. The program stars were selected from a list of MK-classifications of F and G stars by Harlan(1969) and one of the stars was HR7308. The observations of HR7308 started in 1972 without any knowledge of the observations by Breger, but it was not clear that the star was a Cepheid until 1973. The star has been observed every year since the discovery of the Cepheid variation and especially after the discovery of the variations in the amplitude in 1974. In 1978 a computer program for the analysis of the variation of the amplitude was developed. HR7308 was observed by the author with the IUE satellite on March 21, 1980 from Villafranca near Madrid. Radial velocities have been observed at l'observatoire de Haute Provence in October 1979 and April 1981. the preliminary results of the observations 1972-1980 Henriksson(1980) were presented at the I.A.U. Fifth Regional Meeting "Variability in Stars and Galaxies" in Lie'ge 1980. The Cepheid variation of HR7308 was independently discovered by Percy,Baskerville and Trevorrow(1979) but they could not determine a correct period from their observations during the summer 1978. Another independent discovery,resulting in a correct period and knowledge of the decrease in the amplitude during the end of the spring 1978 was made by Burki and Mayor(1980) from variations of the radial velocity.

OBSERVATIONS FROM KVISTABERG 1972-1982

The observations were performed in the UBV system with the 40cm reflector at the Kvistaberg observatory, the outstation of the observatory in Uppsala. During the first years a D.C. amplifier was used ; in August 1975 a Brookdeal 5C1 photon counting system was installed , which led to improved observational accuracy. The photometer is an one-channel system and the observations have been made in the sequence V-B-U. The time of integration was 10 seconds and generally three or four filter sequences were combined to one observation. Normally the comparison star was observed before and after the program star ,but sometimes when the weather was very stable, one observation of each star was considered as sufficient. During nights with many well calibrated standards observed the general zero-point was used, otherwise the zero-point determined by the very well calibrated comparison star has been applied. The photometric observations were reduced using well determined transformations for both V,B-V,and U-B deduced from more than 1000 observations of UBV standard stars. The deviations from the standard values can be completely explained by observational errors. The extinction coefficients were determined for every night if possible and were corrected for mean colour dependence. It is therefore not necessary to use stars of the same spectral type as comparison stars. The criteria for the selection of comparison stars have instead mainly been that they should be close to the program star in the sky and not too faint. This makes it possible to perform the observations very quickly,which of course is very important if the weather is unstable. The comparison stars for HR7308 have been HD180316 (=BD27 3307) and HR7305 = HD180553 (=BD27 3313). HR7305 was mainly observed in the beginning as a check star for HD180316. The result of the UBV observations of these two stars relative to primary UBV standards is given in Table 1. A change of a quartz optical element in the photometer in November 1975 caused a great change in the tranformations especially in the UV. Because of this and the change from D.C. amplifier to photon counting system a few months earlier it is natural to divide the data for the comparison stars into two periods. This gives information about the quality of the observations and the transformations.

Table 1

Star	V	m.e.	s.d.	B-V	m.e.	s.d.	U-B	m.e.	s.d.
First period 1972 September 18 - 1975 November 7									
HD180316	6.910	0.001	0.009	-0.068	0.001	0.012	-0.486	0.002	0.018
HR 7305	6.531	0.004	0.008	-0.080	0.004	0.010	-0.567	0.008	0.016
Second period 1975 November 22 - 1980 May 11									
HD180316	6.912	0.001	0.008	-0.067	0.001	0.006	-0.492	0.001	0.009
The values used for the two comparison stars and the total scatter.									
HD180316	6.911	.0007	.0084	-0.067	.0008	.0091	-0.488	.0011	.0136
HR 7305	6.531	0.004	0.008	-0.080	0.004	0.010	-0.567	0.008	0.016

The data in Table 1 includes 143 observations of HD180316 and only 4 observations of HR7305. More observations have been made after this study of the comparison stars but the values given above have not been changed. If we can make a mathematical model for the variations of the program star HR7308 with standard deviations comparable with the corresponding deviations for the comparison star HD180316 we can consider these as caused by observational errors. The observations of HR7308 are presented in Table 2.

Table 2

H.J.D.-2440000.0	V	B-V	U-B	AIR-MASS
1579.4089	6.121	0.618	0.298	1.528
1579.4193	6.127	0.618	0.313	1.588
1579.4290	6.146	0.611		1.651
1651.3114	6.213	0.618	0.374	2.520
1651.3204	6.199	0.622	0.345	2.691
1695.1851	6.140	0.627	0.335	2.436
1695.1948	6.144	0.624	0.310	2.610
1908.5077	6.125	0.609	0.294	1.539
1908.5181	6.129	0.586	0.295	1.601
1908.5264	6.116	0.607	0.310	1.656
1909.4932	6.182	0.634	0.348	1.478
1909.5036	6.191	0.632	0.362	1.532
1911.4817	6.118	0.602	0.296	1.449
1911.4754	6.116	0.600	0.289	1.421
1912.3370	6.209	0.633	0.364	1.174
1912.3419	6.198	0.646	0.366	1.174
1917.3465	6.152	0.609	0.283	1.179
1919.3779	6.130	0.627	0.302	1.214
1919.3828	6.135	0.635	0.279	1.222
1919.5351	6.173	0.620	0.321	1.988
1919.5428	6.181	0.614	0.308	2.074
1920.3385	6.139	0.611	0.265	1.179
1920.3434	6.153	0.598	0.265	1.182
1920.4459	6.122	0.588	0.307	1.398
1920.5006	6.118	0.583	0.296	1.701
1920.5062	6.113	0.588	0.267	1.743
1920.5484	6.100	0.613	0.303	2.179
1920.5567	6.102	0.619	0.303	2.295
1944.2994	6.105	0.603	0.309	1.199
1944.3057	6.121	0.590	0.302	1.207
1944.4199	6.099	0.618		1.588
2346.3441	6.294	0.663	0.369	1.748
2385.2644	6.255	0.653	0.341	1.995
2395.1877	6.241	0.660	0.355	1.588
2409.1733	6.227	0.649	0.328	1.759
2421.1837	6.182	0.609	0.292	2.225
2660.4538	6.196	0.642	0.340	1.559
2661.3410	6.103	0.579	0.321	1.206
2661.5210	6.093	0.572	0.329	2.187
2663.3536	6.175	0.643	0.359	1.233
2663.5019	6.217	0.662	0.344	2.017
2684.4007	6.223	0.652	0.348	1.627
2685.3253	6.100	0.588	0.325	1.303
2685.4521	6.113	0.610	0.321	2.124
2696.3699	6.212	0.646	0.356	1.636
2704.3179	6.142	0.619	0.344	1.464
2704.3969	6.127	0.605	0.312	2.073
2706.2384	6.112	0.608	0.326	1.233
2724.3234	6.162	0.604	0.324	1.868
2739.3103	6.160	0.621	0.315	2.177
2753.2526	6.188	0.644	0.313	1.950
2755.2305	6.122	0.610	0.294	1.794
2764.2362	6.115	0.606	0.307	2.107
2780.1934	6.194	0.616	0.309	2.124

3008.4076	6.180	0.621	0.291	1.233
3008.4097	6.153	0.614	0.305	1.237
3011.3913	6.164	0.619	0.320	1.219
3015.4042	6.193	0.644	0.321	1.264
3042.4492	6.215	0.633	0.303	1.855
3100.2229	6.182	0.642	0.345	1.412
3100.2298	6.175	0.637		1.442
3102.2389	6.193	0.636	0.314	1.512
3102.2465	6.188	0.619	0.316	1.555
3102.2534	6.186	0.618	0.312	1.596
3102.2583	6.188	0.629	0.305	1.627
3259.5235	6.151	0.606	0.309	1.388
3281.4657	6.092	0.573	0.287	1.377
3282.4810	6.227	0.644	0.332	1.317
3373.4803	6.249	0.650	0.332	1.454
3375.4216	6.063	0.565	0.274	1.272
3375.4244	6.060	0.571	0.274	1.279
3377.4059	6.163	0.638	0.335	1.249
3377.4080	6.158	0.639	0.331	1.254
3386.3812	6.189	0.645	0.328	1.248
3388.3350	6.251	0.669	0.336	1.189
3388.3371	6.249	0.673	0.330	1.191
3388.5200	6.215	0.630	0.308	2.056
3395.3316	6.179	0.640	0.333	1.206
3395.3330	6.178	0.646	0.332	1.207
3397.3124	6.257	0.671	0.328	1.191
3397.3152	6.258	0.667	0.328	1.193
3399.2967	6.057	0.558	0.271	1.182
3399.2994	6.039	0.566	0.272	1.184
3399.4317	6.055	0.580	0.273	1.562
3399.4345	6.047	0.590	0.276	1.579
3399.5363	6.098	0.571	0.306	2.789
3400.3196	6.258	0.655	0.334	1.207
3400.3217	6.254	0.657	0.326	1.210
3453.3614	6.127	0.604	0.346	2.262
3453.3635	6.118	0.616	0.332	2.293
3457.2709	6.119	0.603	0.284	1.535
3457.2737	6.126	0.593	0.283	1.550
3457.3603	6.057	0.575	0.299	2.411
3430.2615	6.148	0.633	0.320	1.942
3457.3568	6.051	0.583	0.273	2.356
3430.2643	6.150	0.631	0.326	1.971
3625.5034	6.284	0.669	0.332	1.465
3625.5193	6.280	0.672		1.396
3625.5221	6.302	0.661	0.348	1.385
3625.5311	6.277	0.677	0.342	1.353
3625.5380	6.272	0.673	0.344	1.330
3625.5401	6.275	0.668	0.321	1.323
3626.5082	6.239	0.669	0.362	1.430
3626.5054	6.250	0.655	0.369	1.442
3626.5103	6.240	0.673	0.358	1.421
3630.5124	6.002	0.553	0.283	1.370
3630.5159	6.013	0.541	0.284	1.357
3756.3678	6.212	0.668	0.333	1.246
3756.3699	6.218	0.658	0.351	1.251
3773.3467	6.104	0.602	0.305	1.305
3773.3483	6.104	0.606	0.305	1.310
3787.4415	6.198	0.628	0.338	2.204
3787.4443	6.188	0.634	0.325	2.242
3807.3669	6.172	0.608	0.309	1.956

3807.3704	6.172	0.613	0.300	1.993
4217.1375	6.194	0.652	0.297	1.519
4217.1910	6.188	0.646	0.300	1.538
4222.2458	6.125	0.615	0.292	2.122
4222.2493	6.111	0.618	0.302	2.166
4369.4800	6.209	0.625	0.308	1.404
4371.4787	6.088	0.593	0.301	1.388
4371.4856	6.094	0.588	0.305	1.362
4399.4320	6.148	0.610		1.292
4399.4341	6.152	0.610		1.286
4483.3176	6.070	0.585	0.296	1.177
4467.4281	6.206	0.656	0.328	1.264
4470.3592	6.183	0.641	0.347	1.180
4513.3864	6.128	0.591	0.340	1.581
4515.3796	6.248	0.648	0.378	1.577
4516.2398	6.092	0.595	0.311	1.182
4516.2474	6.100	0.586	0.303	1.188
4532.2023	6.130	0.590	0.300	1.186
4532.2085	6.130	0.584	0.288	1.192
4535.2074	6.118	0.591	0.290	1.199
4535.2136	6.129	0.580	0.295	1.206
4544.1859	6.106	0.581	0.294	1.202
4545.1887	6.260	0.662	0.361	1.210
4551.2250	6.245	0.659	0.343	1.323
4552.3096	6.188	0.631	0.356	1.786
4552.3110	6.191	0.645	0.335	1.798
4563.2215	6.241	0.664	0.334	1.429
4563.2291	6.245	0.658	0.343	1.463
4569.2390	6.234	0.640	0.321	1.608
4569.2522	6.235	0.623	0.318	1.699
4573.1858	6.183	0.653	0.344	1.394
4579.1674	6.206	0.644	0.358	1.386
4581.2027	6.233	0.629	0.345	1.590
4590.1357	6.246	0.632		1.381
4590.1454	6.232	0.638	0.351	1.420
4590.1551	6.244	0.627	0.335	1.463
4590.1835	6.219	0.635	0.338	1.622
4590.2063	6.220	0.614	0.338	1.792
4590.2375	6.211	0.608	0.323	2.113
4590.2465	6.186	0.612	0.309	2.231
4590.2590	6.180	0.601	0.323	2.420
4607.1504	6.061	0.592	0.296	1.719
4607.1621	6.071	0.603	0.291	1.816
4607.1663	6.086	0.589	0.303	1.853
4607.1767	6.080	0.597	0.302	1.955
4607.1808	6.082	0.595	0.292	2.000
4607.1975	6.090	0.590	0.305	2.203
4607.2009	6.099	0.581	0.296	2.251
4607.2120	6.100	0.589	0.290	2.420
4607.2245	6.106	0.583	0.304	2.644
4607.2342	6.107	0.594	0.308	2.848
4607.2459	6.101	0.592	0.322	3.139
4610.1649	6.037	0.602	0.305	1.920
4611.1632	6.170	0.614	0.291	1.935
4614.7410	6.120	0.613	0.308	1.869
4614.7480	6.105	0.620		1.807
4615.1618	6.263	0.676	0.366	2.037
4618.1718	6.252	0.682	0.358	2.271
4618.1801	6.252	0.706	0.357	2.398
4630.7104	6.097	0.577	0.288	1.749

4691.5401	6.259	0.660	0.328	1.749
4691.5442	6.257	0.658	0.315	1.717
4702.5635	6.058	0.586	0.295	1.427
4702.5704	6.064	0.588	0.284	1.399
4714.5199	6.073	0.577	0.295	1.475
4714.5282	6.063	0.595	0.290	1.436
4720.5192	6.067	0.605	0.303	1.404
4720.5254	6.082	0.606	0.299	1.380
4723.5165	6.100	0.590	0.302	1.383
4728.4861	6.303	0.693	0.376	1.452
4728.4930	6.297	0.698	0.367	1.421
4746.4436	6.352	0.692	0.393	1.421
4747.4073	6.104	0.613	0.340	1.596
4754.4287	6.042	0.545	0.267	1.394
4813.4262	6.291	0.682	0.421	1.185
4813.4317	6.283	0.685	0.419	1.190
4819.4074	6.301	0.723	0.375	1.183
4823.3988	6.062	0.604	0.314	1.184
4836.3578	6.088	0.579	0.274	1.181
4836.3662	6.079	0.576	0.269	1.186
4840.3506	6.321	0.716	0.361	1.183
4840.3562	6.299	0.721	0.369	1.187
4841.3345	6.108	0.619	0.327	1.176
4843.4645	6.308	0.717	0.373	1.494
4843.4715	6.320	0.705	0.363	1.530
4850.4268	6.182	0.660	0.334	1.408
4871.2921	6.171	0.639	0.342	1.206
4895.2790	6.241	0.691	0.353	1.312
4899.3540	6.023	0.587	0.292	1.747
4918.2937	6.147	0.597	0.295	1.673
4935.2418	6.127	0.602	0.316	1.640
4943.1687	6.267	0.666	0.342	1.379
4953.1647	6.128	0.632	0.349	1.480
4956.1676	6.137	0.636	0.327	1.538
4982.1821	6.189	0.626	0.307	2.375
5022.6675	6.140	0.602	0.292	1.536
5055.5989	6.128	0.599	0.306	1.422

OTHER PHOTOELECTRIC MEASUREMENTS

When it was clear that periods of three years and even longer were important in the light variations of HR7308,a search of older observations was made. The oldest published observation of HR7308 in the UBV-system is made by Argue(1963) and is a mean of three measurements. A.N. Argue has kindly put details about his observations at the disposal of the author of this paper. These observations were made during a maximum of the amplitude and they are,therefore, very useful for the determination of the period of the amplitude variation. The individual observations by Argue made at Kitt Peak in April 1962 are given in Table 3.

Table 3

Year	Day	H.J.D.-2400000.0	V	B-V	U-B
1962	April 1	37756.987	6.094	0.595	0.326
	April 15	37770.948	6.276	0.674	0.378
	April 22	37777.932	6.151	0.621	0.366

Possible systematic differences between Argue's and the author's observations are assumed to be negligible, because each series of data is accurately reduced to primary UBV standards.

During a survey for new Delta Scuti stars Breger(1969) discovered that HR7308 showed "semi-regular" variations. The observations, made at the Lick observatory in 1966,1967 and 1969,have now been published by Breger(1981). They are very accurate and have been used by the author of this paper as a check of the Fourier model already developed. The standard deviation diminished with 2.5% when these data were added to the other data published earlier (see below). The transformation to the V-magnitude is

$$V=dV(HR7308-HR7280)+6.330 \qquad (1)$$

Photoelectric observations of HR7308 have,furthermore,been made by Percy,Baskerville and Trevorrow(1979), Percy and Evans(1980) and by Burki and Mayor(1980). Percy et al. made the first observations during the summer 1978 in both V and B and in 1979 only in V. They obtained 16 observations in 1978 and 28 in 1979. The measurements from 1978 are published in the instrumental BV system and must be transformed. Only zero point corrections have been applied,determined from the differences where the observations of HR7308 by Percy et al. overlap with those by Henriksson. The transformations are given as equations (2)-(5).

Kitt Peak in 1978: $V = V(Percy) - 0.034$ (2)

$B-V = (B-V)(Percy) + 0.025$ (3)

Toronto in 1978: $V = V(Percy) + 0.010$ (4)

Kitt Peak in 1979: $V = dV(HR7308-HR7280) + 6.329$ (5)

(One observation from 1978 has been excluded; Percy et al. suspect it to be disturbed by clouds and it deviates also extremely compared to the others).

Burki et al. made their observations in the Geneva system and it is in this case not sufficient to apply only zero-point corrections. Ten observations have been published and seven overlap with the observations by Henriksson. These observations were compared with the corresponding values from the Fourier-model(see below) and linear realations between the systems were derived. The scatter is very small for the colours. The V magnitude only needs a zero-point correction. The transformations from the Geneva system are:

$$V = mv(\text{Geneva}) - 0.010 \quad (6)$$
$$B-V = 0.722\,(B-V)(\text{Geneva}) + 0.759 \quad (7)$$
$$U-B = (U-B)(\text{Geneva}) + 0.3755\,(B-V)(\text{Geneva}) - 1.366 \quad (8)$$

These formulas are ,of course,not valid for stars differing very much from HR7308 in colour and luminosity.

It has also been possible to include three older observations in the Geneva system that were published as a mean in the photometric catalogue by Rufener and kindly communicated in detail by B. Hauck. The same transformations as for the other observations in the Geneva system have been used. These observations are given in Table 4 and are important,since they were made before the first observations by Henriksson. The mv(G) observation on J.D. 1577.358 has not been used been since it has zero weight and deviates +0.03mag from the Fourier model.

Table 4

H.J.D.-2440000.0	mv(G)	(B-V)(G)	(U-B)(G)	V	(B-V)	(U-B)
1224.350	6.039	-0.292	1.753	6.029	0.548	0.277
1555.371	6.106	-0.250	1.769	6.096	0.578	0.309
1577.358	(6.202)	-0.215	1.756	(6.192)	0.604	0.309

THE OLD PHOTOMETRY

A search was made in old photometric catalogues and yielded altogether sixteen measurements or estimates made in the years 1880-1897, for which the dates of observation are accurately recorded. The most important contribution comes from the Harvard Annals 14,44 and 45 by E.C.Pickering(1884, 1899 and 1901) containing visual observations with the meridian photometer and from the Harvard Annal 27 with transformed photographic magnitudes from the plates taken for the HD-Catalogue Pickering(1890). Only two observations come from the Photometrische Durchmusterung by G. Muller and P. Kempf(1899) (but the quality is much higher). It is not trivial to transform these old observations made with different instruments and personal equations to the V magnitude in the UBV-system. Fortunately three observations or more are made with essentially the same system except for the two in the Potsdam photometry. These observations can ,however, be transformed directly by a least-square fit to the difference in magnitude between m(Potsdam) and V observed at Kvistaberg for stars in common. Ten stars(one fundamental star) of different colours have been used and the relation found is

$$V = m(\text{Potsdam}) + 0.019(B-V) - 0.211 \qquad (9)$$

with the standard deviation 0.058mag.

The transformation of HR7308 and the comparison stars are: HR7308: V =m(P)-0.20 , HR7305 and HD180316: V =m(P)-0.21. In order to derive zero-point corrections to the different series of Harvard observations , all these observations were plotted with the phase of the pulsation as abscissa. The pulsation can easily be seen in such a plot even if the scatter is great. Assuming that the mean magnitude of the variable was the same at end of the nineteenth century as it is today , a zero-point correction of +0.20mag to the observations made in 1893, and of -0.05mag to all the other Harvard observations seems appropiate. The modern mean-magnitude for HR7308 is V = 6.16. These old observations have been used below in the determination of the secular variation of the pulsation period and the period of the amplitude variation. The original old data and the different transformations and corrections are given in Table 5.

Table 5

Source	Year	Date	J.D.-2400000	mv	Zero corr.	Night corr.	V	dV (O-C)
H.A. 14	1880	Aug.16	7944.600	6.36	-0.05	-----	6.31	+0.05
	1881	Jul. 6	8268.713	5.86	-0.05	+0.04	5.85	-0.16
	1881	Aug. 2	8295.638	6.26	-0.05	-----	6.21	+0.16
H.A. 14	1881	Jul.23	8285.70	6.4	-0.05	-----	6.35	+0.06
	1882	Nov.30	8780.49	5.6	Great corrections and scatter this night of direct estimation ;not used.			
	1882	Dec.12	8792.48	6.2	-0.05	-----	6.15	-0.02
H.A. 27	1886	Sept.7	10157.532	6.29	-0.05	-----	6.24	+0.13
	1887	May 15	10407.838	5.99	-0.05	-----	5.94	-0.11
	1887	Aug. 3	10487.643	6.39	-0.05	-----	6.34	+0.05
H.A. 44	1893	Jul. 7	12652.710	5.50	+0.20	+0.20	5.90	-0.17
	1893	Jul.20	12665.674	6.00	+0.20	-----	6.20	-0.05
	1893	Jul.29	12674.649	6.10	+0.20	-----	6.30	+0.05
	1893	Sept16	12723.515	6.20	+0.20	-----	6.40	+0.17
H.A. 45	1897	Aug.26	14163.572	6.43	-0.05	-----	6.38	+0.07
P.A.O.13	1890	Nov. 7	11679.271	6.32	-0.20	-0.06	6.06	+0.02
Potsdam	1895	Sept27	13464.405	6.39	-0.20	+0.04	6.23	+0.04

The statistical properties of the residuals are given in Table 6.

Table 6

Source	N	Mean-deviation	Mean-error	Standard deviation
H.A.14,27,45	9	+0.026	0.035	0.105
H.A.44	4	+0.000	0.072	0.145
P.A.O.P. 13	2	+0.030	0.010	0.014
All old	15	+0.019	0.027	0.105

The probable error for the observations in H.A. of HR7308 and the comparison star HR7305 are 0.12 and 0.08 respectively. The p.e. for HR7308 reduces to 0.07 if we subtract the variation. Stephenson(1959) has given a formula for transformations from Harvard Revised Photometry to V-magnitude:

$$V=m(HR) - 0.010 -0.1*(B-V) \qquad (10)$$

This formula is ,however, not very useful for individual stars as Harvard Revised Photometry gives the mean from all the observations made by different visual photometers used in the Harvard photometry without any attempt to transform the observations to a common photometric system. The scatter in the individual observations in the Harvard photometry makes it useless to determine night corrections from only HR7305 because this can increase the error as this star is near the limiting magnitude for the 2" photometer. If night corrections have been applied they are determined from more than five stars observed close to HR7308. The observations from the Potsdam photometry are more accurate and in this case corrections to the nights have been succesfylly determined from the two comparison stars HR7305 and HD180316.

OBSERVATIONS MADE BY THE AUTHOR AT L'OBSERVATOIRE DE HAUTE PROVENCE

Determinations of radial velocity became of obvious interest,when the Cepheid character of HR7308 was revealed,especially so,when the variability of the amplitude was established. Observing time was kindly put at my disposal at l'observatoire de Haute Provence in October 1979 and April 1981. Eight plates with a dispersion of 12Å/mm and two plates with 20Å/mm were secured with the coude' spectrograph at the 152 cm telescope. The plates were measured with the PDS measuring machine at the observatory of Lund and reduced with the HP21mx computer at the Uppsala observatory ,using a program written by the author. Baked IIa-O plates were used and the useful wavelength range is 3400-5200Å. The comparison spectrum is an iron arc and the same lines are used in the stellar spectra to avoid errors in the standard wavelengths. The program automatically identifies the lines and therefore it is possible to use as many as 100-180 lines per scan. The comparison lines are scanned at four different y values on each side of the spectrum and the stellar spectrum is scanned at two y values. (The y axis is perpendicular to the direction of dispersion which is the direction of the x axis.) The wavelength calibration is determined by the method of least squares from the 900-1300 identified line positions in the eight scans of the comparison spectrum. The following formula has been used:

$$\lambda = a0+a1x+a2x^2+a3x^3+a4x^4+a5x^5+a6y+a7y^2+a8xy+a9xy^2 \qquad (11)$$

y is very small in the stellar spectrum and therefore it is not necessary to include the y terms when computing the radial velocities. However, if the y terms are not included in the calibration, we will get a systematic error in the velocity of the star of about 2km/s. The resulting radial velocities are presented in Table 7.

Table 7

H.J.D.-2400000.	Velocity (km/s)	Vr(O-C) (km/s)	Dispersion (Å/mm)	Year
44152.344	-18.60	-0.54	20	1979
44152.428	-18.55	-0.70	12	
44153.326	-13.53	+0.25	12	
44154.496	-13.05	+0.31	12	
44155.314	-17.47	+0.61	12	
44156.463	-15.34	-0.07	12	
44708.586	-25.58	-0.30	20	1981
44708.637	-22.85	-0.03	12	
44709.605	-15.07	+0.88	12	
44710.560	- 7.57	+0.49	12	

The mean-value of the deviations from the Fourier model is -0.02 +-0.21km/s and the standard deviation is 0.51km/s for the observations during the low amplitude phase in 1979. The Fourier model is here dominated by the observations by Burki et al. but also by Percy et al.. During April 1981 when the amplitude was nearly at maximum the mean value of the deviations is +0.26 +-0.26km/s and the standard deviation 0.53km/s. This part of the amplitude curve is also observed by Albitzky and Shajn 1929 with six spectra and the mean of the residuals is +0.03 +-0.27km/s and with the standard deviation 0.67km/s. A greater scatter at maximum amplitude is expected because the Cepheid may also change the amplitude and the phase-shift from one cycle of the amplitude variation to another.

OTHER MODERN OBSERVATIONS OF RADIAL VELOCITIES

The most important observations of the radial velocity of HR7308 are made by Burki and Mayor(1980) and Burki,Mayor and Benz(1982). They have made 246 very accurate observations using the Coravel spectrophotometer at the 1m Geneva reflector at l'-observatoire de Haute Provence. Their observations of HR7308 started at the end of 1977 and have been made in a systematic way since they during the spring of 1978 realized that the velocity was variable and that the amplitude was decreasing. They have kindly sent their latest observations ending in Februari 1981 to me in advance of publication. Unfortunately they could not continue their observations during the spring and summer of 1981 when the latest maximal amplitude occured. Their observations make it possible to analyze the velocities in the same way as the photometry. The standard deviation for these observations is expected to be 0.50km/s and a good model must reproduce the observations almost as good as this.

Other modern observations of radial velocities that are used in this paper are made by Percy and Evans(1980). They have observed HR7308 with the cassegrain spectrograph on the 1.88-m telescope at the David Dunlop observatory and obtained nine plates during 1979. The published measurements have been corrected to the I.A.U. standard system by adding +1km/s.

OLD RADIAL VELOCITIES

The earliest radial velocity observations of HR7308 are made by Albitzky and Shajn(1933). They have made the observations at the Simeis observatory in Crimea,belonging to the observatory of Pulkovo. The spectrograph was put on a 1 m reflektor and had a dispersion of 36Å/mm. It is very important that they have published the individual observations and carefully calibrated the velocities to the Lick-system as a function of spectral type. The corrections are typically smaller than 1 km/s and the quality of the measurements is very good as can be seen in Table 8. The author of this paper has earlier used data from the same paper and found that the quality is unexpectedly good. A correction of +1km/s has been added to the observations by Albitzky and Shajn as recommended by Abt and Levy(1970). These old radial velocities are very important for the interpretation of HR7308 and therefore they are quoted here in Table 8 after correction by +1km/s. The residuals from the Fourier model are also given.

Table 8

H.J.D.-2400000.	Velocity (km/s)	Vr(O-C) (km/s)	Dispersion (Å/mm)	Year
25790.416	-17.4	+0.34	36	1929
25790.456	-18.9	+0.15		
25798.430	- 9.6	-0.79		
25798.470	- 8.0	-0.52		
25890.449	-23.0	-0.11		
25890.489	-21.7	+1.10		
26182.387	-19.5	-0.33	36	1930

In the original data by Albitzky and Shajn the epochs for two observations made during the same night on the same plate are given as a mean epoch. With fresh silvered mirrors the exposure of one hour gives a spectrum of stars of the magnitude 6.3 in average atmospheric conditions. The author of this paper has therefore tried to reconstruct the real epochs by a correction to the mean epoch of +-0.02 days which is +-0.5hours. This reduces the residuals significantly and does not introduce any systematic error.

There also exist four observations made by Young(1939) at David Dunlop Observatory but he has not published the individual observations because the variation of the velocity was small in his opinion. These observations were made between June 1935 and March 1939. The mean value of the four measurements is -13.4km/s and the p.e. of the mean is 1.2km/s which indicates that the amplitude was low. These observations should also be corrected by adding +1km/s giving -12.4 km/s as the mean. The small p.e. can be easily be explained if the observations were made during the summer 1935 or the summer 1938.

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THE PULSATIONAL PERIOD

When it was clear from the observations made in 1972 that HR7308 is a variable star a systematic photometric investigation started in the autumn of 1973. The first determination of the period for HR7308 was made in 1973 simply by plotting the observations as a function of time. It was soon clear that the period was close to 1.5d and that the semi-amplitude was about 0.04mag.

During 1973 and 1974 a period-testing program was developed. For a sequence of test periods,the program arranges the observations in phase order and sums up the squares of the deviation of every observation from a straight line through the two observations adjacent in phase. The first result for the period of HR7308 was 1.491d and the shape of the light-curve was sinusoidal. The star was classified as a population I Cepheid and the period was the shortest known for its type.

However, as has been pointed out above, the observations from 1974 differed very much from the expected light-curve and the reason was not clear. In 1975 the observations seemed to fit the curve from 1973 reasonably well, and a new determination of the pulsational period was made, excluding the observations from 1974. A computer program for Fourier analysis of the observed light curve was now developed , including a possibility to subtract the influence of a given period from the observations. The effect of the pulsational period 1.490729d was subtracted from all the observations (except those from 1974) and a test for a possible overtone was made. The most significant period was found to be 0.597509d. This result was not very satisfactory, because this period could not be a normal overtone for the Cepheid variation and, even worse the period was the perfect resonance period between the earlier subtracted period and the siderial day. The difference was in fact only 1.4s. When more observations were obtained in 1976 it became clear that not even the period was constant. A possibility to subtract the effect on the times of observation caused by a possible motion of the Cepheid in a circular orbit was therefore included in the program for Fourier analysis. Several "orbital" periods were tested between 1 and 10 years. The smallest scatter was found with the "orbital" period equal to 5+-2 years. If the period change was periodic and the deviations observed at the end of 1974 were associated with the change in the period we could expect to observe the same phenomenon perhaps already at the end of 1977. HR7308 was therefore observed in April and May that year and in fact a small increase of the amplitude was seen. During the summer the nights are too bright at Kvistaberg,so the observations were not started again until August. Now it became completely clear that the amplitude had increased and that the same explaination applied to the observations from 1974.

THE PERIOD OF THE AMPLITUDE VARIATION

When the list of new low amplitude variable stars was published Henriksson(1977) HR7308 was not included ,because the variations of this star were not enough understood at that time.

After the maximum amplitude was reached once again during the spring of 1978 it was possible to estimate the period for the amplitude variation,if this was periodic. The first estimations gave periods around 3.3 years. It is now easy to understand the earlier found resonance with respect to the siderial day as caused by an unsuccessful subtraction of the pulsations ; the amplitude was not constant, even if the observations from 1974 were excluded.

To be able to test if the amplitude variation really is periodic a search for older observations was made. As is mentioned above 15 useful photometric observations made in 1880-1897 and 7 radial velocities observed in 1929-1930 were found with published epochs. The amplitude seemed to be at maximum in 1893 and in 1929 and the time-intervals to 1978 were simply divided by integers to find a common period close to 3.3 years. At that time no modern radial velocity observations were available; (the radial velocity amplitude in 1929 was,however,too great to be at minimum when compared with other small amplitude Cepheids) so the period of the amplitude variation was mainly determined from the photometry. The period 1185d or 3.244years was found to satisfy both the photometry and the radial velocities from 1929-1930 reasonably well.

ANALYSIS OF THE VARIABILITY

In the spring of 1978 a computer program, simply called HR7308,was developed to analyze the observations of HR7308. The starting point was to describe the observed magnitude of the star as a product of two Fourier series, V = A*B, where A describes the variation of the amplitude(period Pa about 3.24years) and B the shape of the light curve(period PC about 1.5days).

A = (a0+a1cosx+a2sinx+a3cos2x+a4sin2x+.....) (15)
B = (b0+b1cosy+b2siny+b3cos2y+b4sin2y+.....) (16)

where x=t/Pa*2pi and y=t/PC*2pi ,t representing the epoch of observation. The observations are fitted by the method of least squares to the completed multiplication of the series A with B , regarding all coefficients of the trigonometric functions as independent variables. If the number of terms in A and B is m and n respectively, m*n independent unknowns,are thus, introduced. This general expression, which simply should be regarded as an interpolation formula representing the observations, takes into account a change of the amplitude and shape of the light curve with the period Pa or,which is equivalent, a change of the amplitude and the period PC with the period Pa. In practice the shape of the light-curve has been kept constant and approximated by a simple cosine curve, because otherwise the solution easily becomes unstable, as the data are not well distributed in phase(the period PC being very close to 1.5 days). The values of m used in the computations are three or four and this seems to be the practical upper limit with the existing data.

The result of the least-squares solution is now used to compute the magnitude at maximum and minimum and the difference in time between the time for maximum and minimum and the corresponding epoch computed from a constant period. One hundred equally spaced values are computed for a Pa cycle and the variation of the amplitude is approximated directly with the formula (15) for A. A similar calculation is made for the difference in time computed above as

C =(c0+c1cosx+c2sinx+c3cos2x+c4sin2x+.....) (17)

(C is,thus, the observed-computed epoch of maximum or minimum). The contribution to the light variation from the pulsations with variable amplitude is now subtracted from the observed magnitudes and is given by A*(b1cosy + b2siny). The variation of the mean magnitude during a Pa cycle is then approximated by

D = (d0+d1cosx+d2sinx+d3cos2x+d4sin2x.....) (18)

To improve the solution,two or three iterations may be nescessary for each pair of values of Pa and PC and are made by a new fit to V = A*B but with the epochs of observation "corrected" by C. The convergence is very rapid and the standard deviations are nearly identical for the first general solution and the explicit solution describing the observations as the light-curve (B) with variable amplitude (A),mean magnitude (D) and period PC (causing the time difference (C)). It is not possible to include the distortion of the light-curve,i.e. the second-order terms cos2y

and sin2y, in this first solution because the data are not well distributed with respect to Pa and PC phases, However,the information about the distortion can be computed from the residuals of the first cosine solution. In the program there is a loop outside the above solution and the period PC is just changed to 0.5*PC for the investigation of the second order terms cos2y and sin2y. After two iterations the second order solution has converged and a new solution for cosy and siny is obtained after three iterations. Finally a new second order solution is determined by three iterations as for the first order. When more data has been included in the computations (about 360 photometric observations) a more complete coverage of the two-dimensional phase-space for Pa and PC was obtained. This makes it necessary and possible to include also the third order terms in (B) for the solution of the photometry. For the third order analysis of the light curve the residuals of the first and second orders are analysed with the period PC changed to 0.333333*PC. After that a final solution is obtained for PC with the 0.5*PC and 0.333333*PC curves removed from the observations , for 0.5*PC with the PC and 0.333333*PC curves removed , and for 0.333333*PC with the PC and 0.5*PC curves removed. With this technique it is possible to describe the light variation as a cosine curve with variable amplitude,distortion,period and mean magnitude. Optimal values of the periods Pa and PC can be determined by studying the dispersion of the residuals between the observed magnitudes and the model for a series of values of Pa and PC.

At the I.A.U. Regional meeting 1980 at Liege the analysis of more than 150 photoelectric observations were presented with the second order approximation of A,C and D and no distortion of the cosine curve. A plot of the variation of the amplitude in V,B-V and U-B ,the observed-computed epoch and the variation of the mean V magnitude was presented Henriksson(1980). After that meeting the photoelectric observations made by Percy et al. and Burki et al. and many good radial velocities observed by Burki et al. with the Coravel became avaible. Unfortunately the radial velocity observations covered only 3/4 of the Pa interval and the important part near the maximum amplitude was completely missing. Unsuccesful and confusing attempts to make an independent analysis of the radial velocity data have delayed the publication of this paper.

When more photometric data from observations during the autumn 1980 were added it became obvious that this first solution was not detailed enough to describe the light variations of HR7308 with reasonable accuracy. The distortion of the cosine curve and a possibility to approximate A,C and D also by a third or fourth order series were included but late in 1980 new problems showed up that could not be solved without the introduction of a new function that corrected for an earlier unknown change of PC. This effect and a secular variation(discussed below) are removed before the optimal values of Pa in Table 9 were computed. The values of Pa in Table 9 are determined from a search for the smallest standard deviation as in other determinations of periods in this paper.

Table 9

Data	Order of B	Pa(days)
Old+modern photometry(1880-1982)	2	1178.5
Old+modern radial velocities(1929-1981)	2(with P4)	1178.6
Old+modern radial velocities(1929-1982)	2(without P4)	1187.0
Modern photometry(1962-1982)	1	1195.0
Modern photometry(1962-1982)	2	1193.0
Modern photometry(1962-1982)	3	1184.0

The meaning of P4 is explained below. The old photometry consists of 15 and the modern of 356 observations. The number of old radial velocities is 7 and the number of modern radial velocities is about 260.

VARIATIONS OF THE PULSATIONAL PERIOD

During the analysis of PC it became obvious that PC has both a secular variation and a variation with a period longer than Pa. This variation was studied for different integer multiples of Pa because no changes in the computer program were needed. For this study the observations by Argue from 1962 were very useful. It was found that the available photometric data was best fitted by a period equal to 5*Pa. The 5*Pa solution worked well until the observations from the summer 1981 were included. It was now clear that 5*Pa was too long a period and that 2.5*Pa was a much better value. The determination of the secular variation is discussed below. When the first models of the radial velocities had been computed and a scan for an optimal value of Pa and PC were made, there was too great a difference between the value of PC from the old and modern photometry and the old and modern velocities. Different values of PC for epochs corresponding to the mean of the interval are listed in Table 10. No correction for the variation of PC is made.

Table 10

Data	PC(days)
Old photometry(1880-1897)	1.490672
Modern photometry (1972-1980)	1.490734
Old+modern photometry (1880-1980)	1.490746
Old+modern radialvelocities(1929-1980)	1.490768

During the short interval for the modern photometry the 2.5*Pa period is more important than the secular variation which, however, dominates for longer intervals. The effect of the secular variation and the "2.5*Pa" variation are subtracted from the observed epochs before the Fourier analysis is made.

In the latest version of the program it is also possible to make a search for an optimal value of the "2.5*Pa" period and to analyse the shape of the Pa curve with time simultaneosly. The optimal value for this period from now on called Pt is found to be close to 2650d from the photoelectric photometry now also including the data by Breger from 1966,67 and 69 and those by Percy and Evans from 1980. The value Pt=2655d has been used because it is equal to 2.25*Pa and is better for the old radial velocities. Before the value of Pt was changed from 2.5*Pa to 2.25*Pa there existed an unexplained difference between the value of PC at J.D. 2443600 from the old and modern photometry and the corresponding radial velocities even when the time was corrected for the secular variation and the Pt effect. The value of PC at J.D. 2443600 from the old+modern photometry is 1.490795d but for the radial velocities it was 1.490790d and a use of 1.490795d made a great increase in the scatter. This problem is now solved by the value Pt = 2655d (there still exists a zero point problem for the old velocities). The stability of this period seems clear because it persists during the 20 years covered by photoelectric photometry.

SECULAR VARIATION OF THE PULSATIONAL PERIOD

If two series of observations are separated by an interval T1 and T2 is another similar interval, and the number of cycles during T1 and T2 are N1 and N2 respectively we can compute the corresponding "mean periods" P1 and P2. If we now assume that the variation of the period is linear with the coefficient a and the constant P0 we get:

$$T1=N1*P1=P0*(N1+a*0.5*N1*(N1+1))=P0*N1*(1+0.5*a*(N1+1))$$
$$T2=N2*P2=P0*(N2+a*0.5*N2*(N2+1))=P0*N2*(1+0.5*a*(N2+1)) \quad (19)$$

because $T1=P0+a*P0+P0+a*2*P0+....P0+a*N1*P0=$

$$=P0*(N1+a*\sum_{N=1}^{N1} N)=P0*(N1+0.5*a*N1*(N1+1))$$

and similary for T2.

$$P1=P0+P0*0.5*a*(N1+1)$$
$$P2=P0+P0*0.5*a*(N2+1) \quad (20)$$

If we solve for a we get;

$$a = 2.*(P2-P1)/(P1*(N2+1)-P2*(N1+1)) \quad (21)$$

PN=P0*(1+a*0.5*(N+1)) is the mean period after N cycles.

The observed-computed(O-C) time difference from constant period P0 is

$$dT(O-C)=T*(1-1/(1+0.5*a*(T/P0+1))) \quad (22)$$

If the observed epochs are "corrected" with dT it is possible to use a constant period P0 to compute all the phases.

For HR7308 the values of P1 and P2 have been determined from the old+modern photometry and radial velocities respectively. The modern photometry includes in this case only the observations by Henriksson and the modern radial velocity observations are made by Burki et. al. and by Henriksson.

P1=1.490746d (photometry)

P2=1.490768d (radial velocity)

N1=(1888-1977)/1.490746*365.25=-21806 (photometry)

N2=(1929-1979)/1.490768*365.25=-12250 (radial velocity)

This gives a=3.09*10-9/cycle=1.13*10-6d/year and P0=1.490796d at 1978. If we want to get the value of P=1.490672d for the years 1880-1397 that has been found from the period testing program of the old data alone we must use a=1.36*10-6d/year if P0=1.490795d at J.D. 2443600. Different combinations of the coefficients a and b in the formula have been tested in order to optimize the

photometry and radialvelocity observations. The final formula for the Cepheid pulsations of HR7308 is,

PC = 1.490795 + 1.22*exp(-6)*(J.D.-2443600)/365.25 days (23)

and PC means in this case the actual period at a given epoch. Formula (23) gives PC=1.490684d at the epoch of the old photometry.

To further improve the existing model it is necessary to take into account the possibility that the shape and maximum value of the amplitude variation may not be exactly repeated from one amplitude cycle to another. The residuals from the photometry made by Argue in 1962 and especially by the author in 1974 are greater than expected and are systematic in such a way that a somewhat smaller maximum amplitude would decrease the deviations. For the modern radial velocities by Burki et al. , Percy Evans and Henriksson there cannot exist such a problem because only one Pa cycle is covered by the observations.

THE "FOURTH PERIOD"

The radial velocities measured by Albitzky and Shajn in 1929-1930 do not satisfy the above solutions very well;there remains a zero-point difference of +5km/s for the 1929 observations and of -3km/s for the 1930 observations. Systematic errors of this order of magnitude are very unlikely, because velocities measured by Albitzky and Shajn for other stars compare very well with modern data. The fit to the old radial velocities can be made considerably better, if a fourth periodicity, P4 is allowed in the radial velocities. A fourth period,about 100 days shorter than Pa is discussed below. In order to test the influence of Pt on this problem a scan to determine the value of Pa has been made without the correction for Pt. The data from different scans are shown in Table 11.

Table 11

Photometry(1962-1981)			Radial velocities(1929-1981)	
Pa (days)	Sigma (1:order)	Sigma (2:order)	Sigma (1:order)	Sigma (2:order)
1270	0.01864	0.01763		
1260	0.01781	0.01687		
1250	0.01729	0.01614		
1240	0.01650	0.01509		
1230	0.01613	0.01467		
1220	0.01614	0.01400		
1210	0.01596	0.01372	0.9658	0.8088
1205			0.9422	0.7772
1203			0.9415	0.7749
1202			0.9379	0.7725
1201			0.9352	0.7690
1200	0.01560	0.01347	0.9351	0.7659
1199			0.9397	0.7678
1195			0.9583	0.8177
1192			0.9407	0.8075
1191			0.9337	0.8034
1190	0.01559	0.01350	0.9308	0.8013
1189			0.9328	0.8027
1188			0.9370	0.8068
1185			0.9557	0.8364
1180	0.01616	0.01392	1.0049	0.8946
1175			0.9976	0.8949
1170	0.01672	0.01437		
1160	0.01691	0.01454		
1150	0.01712			

Sigma=Standard deviation

There is no problem to find a unique value of Pa from the photometry even if Pt is not included in the analysis. For the old+modern radial velocities the situation is more problematic. Even if the smallest standard deviations can be found for Pa=1200d this period is not useful because the deviations for the old velocities are not very likely distributed. The other minimum with Pa=1190d seems more reasonable even if the standard deviation is greater but it is obvious that we can not get a re-

liable value of Pa from the velocities. The need for a fourth period is not introduced by a wrong value of Pt. Because the introduction of Pt has no significant effect on the determination of Pa from the old+modern photometry this value can be considered as the most certain. The value of Pa determined from the modern photometry is more dependent on Pt because the interval covered is comparable with Pt. The determination of Pa from the modern photometry and with the effect of Pt=2655d included gives Pa=1134d which can be seen in Fig. 1. To get an optimal value of Pa from the old+modern photometry the effect of Pt on the observed epochs is also removed in the scan presented in Table 12.

Table 12

Pa(days)	Sigma
1180.00	0.02402
1179.50	0.02399
1179.25	0.02398
1179.00	0.02396
1178.75	0.02396
1178.50	0.02395
1178.25	0.02395
1178.00	0.02400
1177.75	0.02403

Pa 1178.5 days has been chosen as the optimal photometric value from Table 12. The importence of Pt in the determination of Pa from the modern photometry is to give a value that is much closer to the best determination from the old+modern photometry. That Pt is a unique and important period can also be understood if we remember the first discovery of the timeshifts that were interpreted as an orbital motion. At that time only the period PC was used because Pa was unknown. The 5 year period found can be explained as a combination of the timeshifts with period Pa=3.23years and Pt=7.27years.

A search for the optimal value of the fourth period mentioned above, from now on called P4, with Pa=1180d and Pt=2655d is presented in Table 13. The data are all the known radial velocities.

Table 13

P4(day)	Sigma(km/s) (2:order)
3020	0.5637
2588	0.5611
2265	0.5575
2013	0.5540
1812	0.5501
1647	0.5463
1510	0.5428
1394	0.5387
1294	0.5322
1208	0.5236
1132	0.5169
1070	0.5199
1013	0.5354

From Table 14 optimal combinations of Pa and P4 can be determined for all the radial velocities. The epochs are corrected with Pt=2655d.

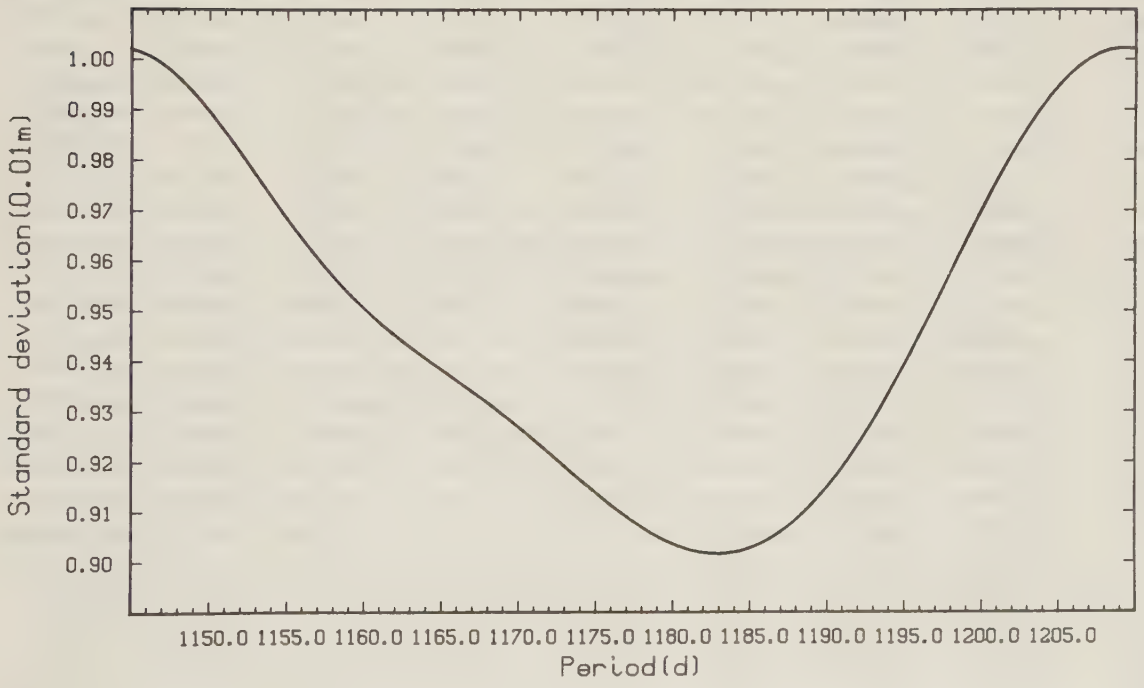

Fig. 1

Table 14

Pa(days) P4(days)	1183	1182	1181	1180	1179	1178	1177	1176
1140	0.4827	0.4886	0.4863	0.4794	0.4741	0.4763	0.4851	0.4870
1135	0.4840	0.4891	0.4862	0.4789	0.4746	0.4757	0.4845	0.4867
1130	0.4864	0.4902	0.4864	0.4784	0.4726	0.4747	0.4837	0.4864
1129	0.4871	0.4906	0.4865	0.4783	0.4724	0.4745	0.4837	0.4863
1128	0.4879	0.4910	0.4867	0.4786	0.4721	0.4743	0.4834	0.4862
1127	0.4888	0.4915	0.4869	0.4782	0.4720	0.4740	0.4834	0.4861
1126	0.4900	0.4921	0.4871	0.4785	0.4718	0.4738	0.4831	0.4860
1125	0.4914	0.4929	0.4878	0.4782	0.4715	0.4735	0.4830	0.4859
1124	0.4931	0.4940	0.4879	0.4783	0.4713	0.4732	0.4826	0.4858
1123	0.4953	0.4950	0.4885	0.4783	0.4711	0.4730	0.4824	0.4858
1122	0.4980	0.4966	0.4892	0.4786	0.4708	0.4727	0.4822	0.4857
1121	0.5016	0.4988	0.4903	0.4790	0.4707	0.4724	0.4820	0.4856
1120	0.5063	0.5016	0.4919	0.4795	0.4705	0.4720	0.4818	0.4856
1119	0.5131	0.5057	0.4940	0.4804	0.4705	0.4718	0.4815	0.4855
1118	0.5227	0.5113	0.4974	0.4797	0.4707	0.4715	0.4813	0.4856
1117	0.5365	0.5207	0.5024	0.4918	0.4712	0.4714	0.4812	0.4854
1116	0.5583	0.5343	0.5102	0.4854	0.4723	0.4715	0.4812	0.4856
1115	0.5911	0.5561	0.5235	0.4922	0.4747	0.4720	0.4814	0.4854
1112	0.8103	0.7950	0.6941	0.5881	0.5147	0.4854	0.4840	0.4891
1111	0.7470	0.8088	0.8256	0.7230	0.5816	0.5081	0.4920	0.4910
1110	0.6504	0.7172	0.7821	0.8413	0.7557	0.5792	0.5151	0.5000
1109	0.5836	0.6316	0.6788	0.7400	0.8320	0.8002	0.5919	0.5247
1108	0.5462	0.5824	0.6102	0.6388	0.6887	0.8468	0.8223	0.6100
1107	0.5258	0.5533	0.5732	0.5872	0.6044	0.6402	0.7419	0.8436
1106	0.5138	0.5359	0.5504	0.5594	0.5648	0.5751	0.6051	0.6934
1100	0.4899	0.5016	0.5057	0.5045	0.5032	0.5058	0.5112	0.5128
1090	0.4864	0.4960	0.4948	0.4908	0.4872	0.4893	0.4966	0.4967
1081	0.4887	0.4947	0.4929	0.4870	0.4826	0.4846	0.4923	0.4934
1080	0.4892	0.4948	0.4928	0.4867	0.4822	0.4842	0.4920	0.4932
1079	0.4897	0.4950	0.4928	0.4865	0.4819	0.4839	0.4917	0.4929
1078	0.4903	0.4952	0.4928	0.4863	0.4816	0.4835	0.4914	0.4927
1077	0.4909	0.4955	0.4928	0.4861	0.4813	0.4832	0.4911	0.4925
1076	0.4915	0.4959	0.4929	0.4859	0.4809	0.4829	0.4908	0.4923
1075	0.4923	0.4963	0.4930	0.4858	0.4806	0.4826	0.4906	0.4922
1074	0.4931	0.4967	0.4931	0.4857	0.4804	0.4823	0.4903	0.4920
1073	0.4940	0.4970	0.4933	0.4856	0.4801	0.4820	0.4900	0.4918
1072	0.4951	0.4977	0.4936	0.4857	0.4799	0.4818	0.4898	0.4916
1071	0.4962	0.4983	0.4939	0.4857	0.4797	0.4815	0.4895	0.4914
1070	0.4974	0.4991	0.4942	0.4858	0.4795	0.4812	0.4893	0.4913
1069	0.4988	0.5000	0.4947	0.4859	0.4793	0.4810	0.4890	0.4911
1068	0.5004	0.5010	0.4953	0.4860	0.4793	0.4807	0.4888	0.4910
1067	0.5025	0.5022	0.4961	0.4863	0.4791	0.4805	0.4886	0.4908
1066	0.5046	0.5037	0.4970	0.4866	0.4790	0.4803	0.4883	0.4907
1065	0.5070	0.5053	0.4980	0.4870	0.4790	0.4801	0.4881	0.4905
1064	0.5093	0.5070	0.4992	0.4875	0.4791	0.4799	0.4879	0.4904
1063	0.5131	0.5094	0.5007	0.4883	0.4792	0.4798	0.4877	0.4902
1062	0.5168	0.5125	0.5025	0.4892	0.4795	0.4797	0.4875	0.4901
1061	0.5214	0.5159	0.5049	0.4907	0.4799	0.4797	0.4874	0.4900
1060	0.5270	0.5205	0.5079	0.4924	0.4803	0.4798	0.4873	0.4900
1059	0.5343	0.5259	0.5119	0.4947	0.4818	0.4800	0.4873	0.4900
1057	0.5545	0.5419	0.5231	0.5002	0.4853	0.4814	0.4876	0.4902
1055	0.5860	0.5699	0.5445	0.5148	0.4933	0.4849	0.4891	0.4914
1053	0.6426	0.6152	0.5866	0.5490	0.5125	0.4945	0.4934	0.4938
1051	0.7361	0.7095	0.6765	0.6235	0.5645	0.5226	0.5067	0.4976

It is very interesting to see that when the old+modern radial velocities are corrected for the effect of Pt and P4 the optimal value of Pa is 1178.6d a value that is not significantly different from Pa=1178.5d from the old+modern photometry. Pa in Table 14 is not sensitive to the different values of P4. The value of Pa=1178.5 will be adopted as the best result. There exist two minima for P4 in Table 14, 1119.0d and 1063.5d and the standard deviation is somewhat smaller for 1119.0d. The difference is not important and therefore both possibilities have been studied. Both standard deviations are very close to the estimated observational errors. The value 1119.0d is very close to 1121.0d which is the mean of 1178.5 and 1063.5d. If we choose P4=1063.5d we can find a lot of interesting correlations between Pa,Pt and P4 and therefore the value of P4 from now on will be P4=1063.5d. It seems impossible to avoid the introduction of P4 in the analysis of the old+modern radial velocities.

THE FOURIER MODEL

The photometric and radial velocity observations are presented in this chapter as diagrams and tables with the Fourier coefficients. The abscissas are 'Phase' and the unit is Pa=1178.5day with phase=0.0 at J.D.=2443597.7 wich is the epoch for the maximum amplitude for the V magnitude. The epoch for the Fourier analysis is J.D.=2443600.0 which is important to remember when the Fourier coefficients are used.

In Table 15 the epochs for maximum brightness in V,B-V and U-B and the maximal negativ velocity for the Cepheid during the pulsational cycle are given for the cosx component. Other parameters in Table 15 are the epoch for maximum amplitude, the maximum amplitude,the number of observations used and the standard deviation for the model.

Table 15

Quantity	Cepheid max. J.D.-2440000	Brightest max. J.D.-2440000	Maximum amplitude	N	Standard deviation
V(cosx)	3600.693	3597.7	0.159	356	0.0099
V(cos2x)	3600.558	3596.5	0.022	356	0.0099
V(cos3x)	3600.441	3569.8	0.007	356	0.0099
V(cosx)	3600.694	3597.3	0.161	356	0.0103
V(cos2x)	3600.548	3588.8	0.024	356	0.0103
B-V (cosx)	3600.667	3608.7	0.074	237	0.0104
B-V (cos2x)	3600.426	3675.8	0.012	237	0.0104
U-B (cosx)	3600.596	3581.2	0.061	218	0.0123
U-B (cos2x)	3600.474	4132.7	0.021	218	0.0123
R.V.(cosx)	3600.701	3571.3	12.35km/s	260	0.476km/s
R.V.(cos2x)	3600.505	3540.4	1.52km/s	260	0.476km/s

If the possibility to correlate the amplitude variation with Pt=2655d is utilized for the V observations(third order,first model in Table 15) the standard deviation is only 0.0090m. The standard deviation for the V observations of the most observed comparison HD180316(Table 1) is 0.0084 which shows that this model for HR7308 almost describes the observations as well as the observational errors admit. This final model for the variation of the V magnitude can be found in Fig.2a and 2b. In Fig.2a the variation of the V amplitude is presented as three curves;the thick curve representing cosx,the thin curve cos2x and the dotted curve cos3x respectively. The amplitude for cosx is given as a fourth order Fourier series and the amplitude for cos2x and cos3x as second order series. The Fourier coefficients are given in Table 16.

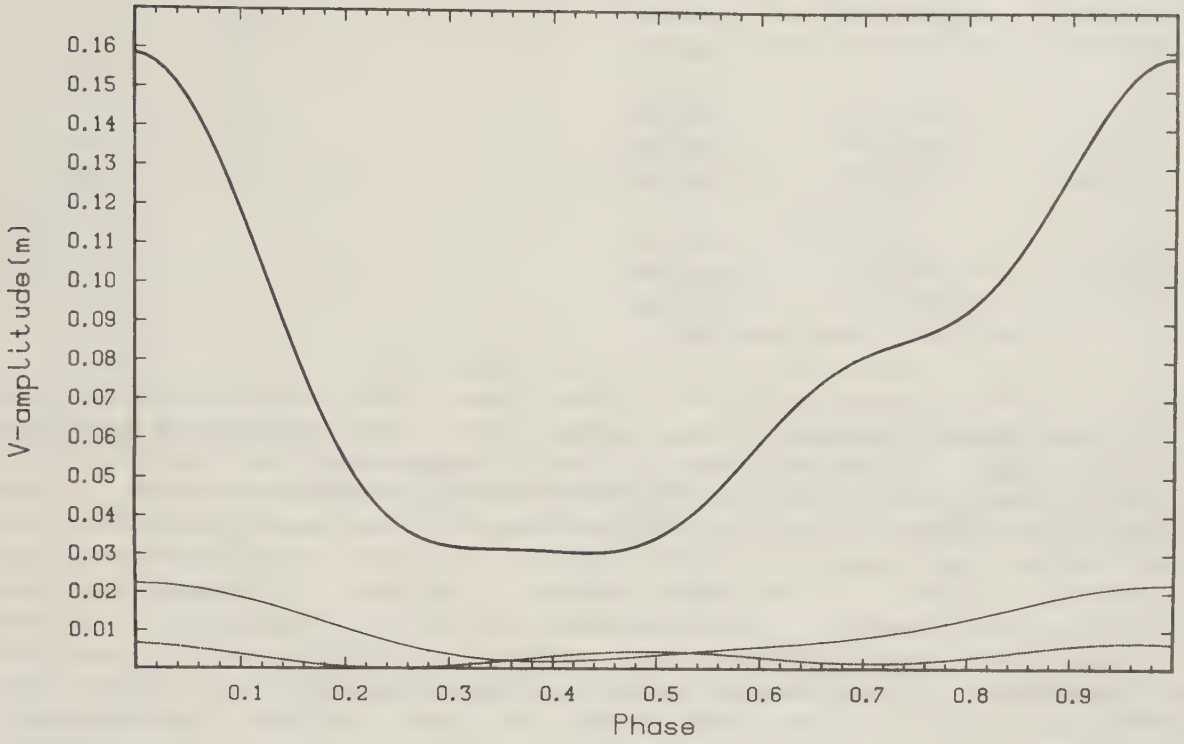
V-amplitude(m)
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Phase

Fig. 2a

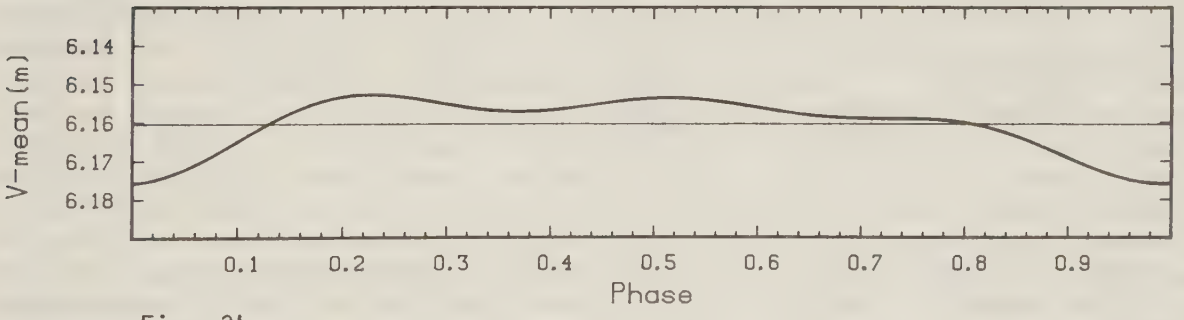
V-mean(m)
6.14
6.15
6.16
6.17
6.18
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Phase

Fig. 2b

Table 16

A(m)	B(m)	N	Curve
0.079143	0.000000	0	cosx
0.051855	-0.023272	1	cosx
0.017451	0.003376	2	cosx
0.009762	0.001898	3	cosx
0.000969	0.000706	4	cosx
0.010662	0.000000	0	cos2x
0.009267	-0.002213	1	cos2x
0.002298	0.000677	2	cos2x
0.003265	0.000000	0	cos3x
0.001044	-0.000855	1	cos3x
0.002235	-0.000853	2	cos3x

All the curves have maximum at the same time and the cosx and cos2x have also minimum at the same time. The secondary maximum for cos3x may (if significant) be a compensation for 'Fourier noise' in the cosx curve at a phase where few data exist. The 'secondary' bends on the rising branch of the cosx curve may indicate that the shape and amplitude is not the same from one cycle to the other. The data comes from 5-6 different cycles which are only partly covered and the resulting shape of the curve may be a compromise where parts from different cycles are put together. The radial velocity amplitude(Fig.4) does not show these bends and the reason may be that the data comes mostly from one cycle. However,the shape and amplitude seem to vary in a periodic way within 10% of the amplitude. A significant feature is the very rapid decrease of the amplitude and the slow increase. The decrease takes place during phase 0-0.25 and then the amplitude is nearly constant during phase 0.25-0.50 and after that during phase 0.50-1.00 the amplitude is clearly increasing. The radial velocity amplitude (Fig.4) is increasing also during phase 0.25-0.50 even if it is not as steep as during phase 0.50-1.00 . The assymetric shape of the amplitude variation excludes the possibilty that the variation is caused by two superimposed pulsational periods with nearly the same period. From a study of Fig.2a it can also be found that the devitation of the Cepheid curve from a cosine curve is 5% at minimum amplitude but increases to about 15% at maximum amplitude. In Fig.2b the variation of the mean magnitude for the pulsational cycles is illustrated. During the maximum amplitude the V mean drops 0.02mag. The V mean is negatively correlated with the amplitude and reaches its maximum close to phase 0.25 and is then decreasing first slowly and after phase 0.80 more steeply. The fourth order Fourier series that describes the variation of V mean can be found in Table 17.

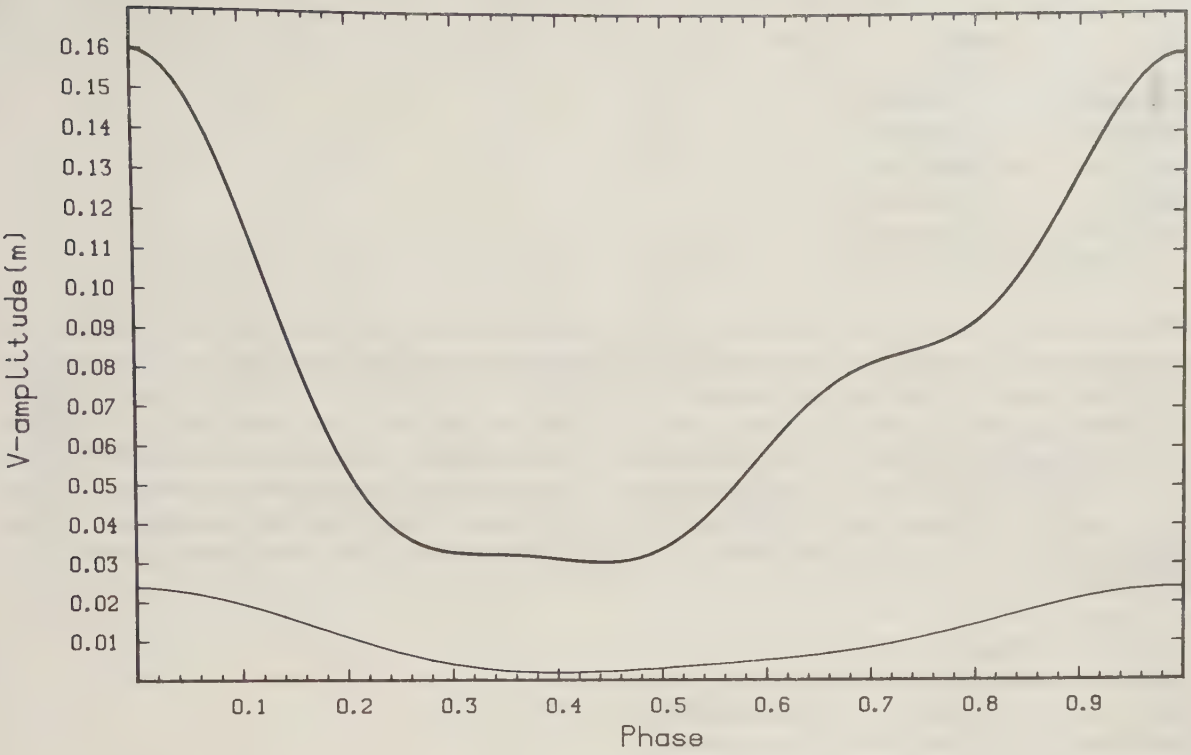

Fig. 3a

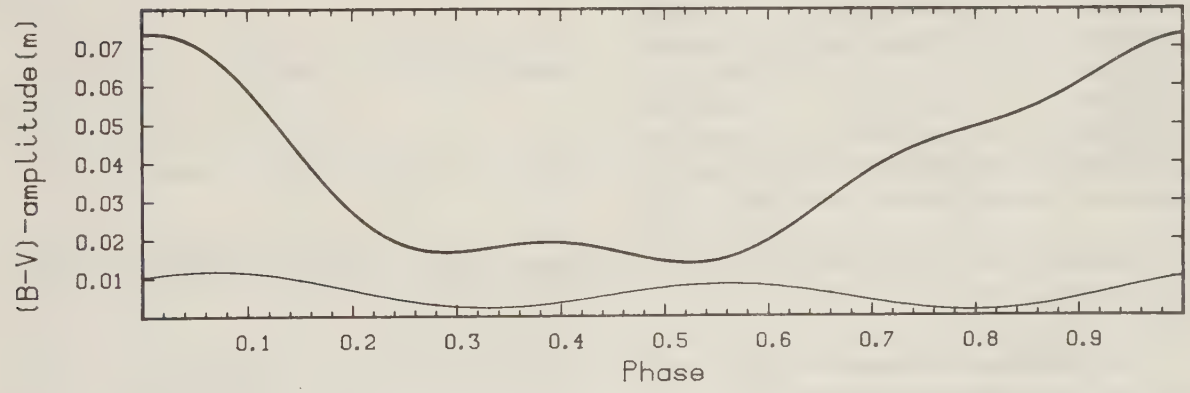

Fig. 3b

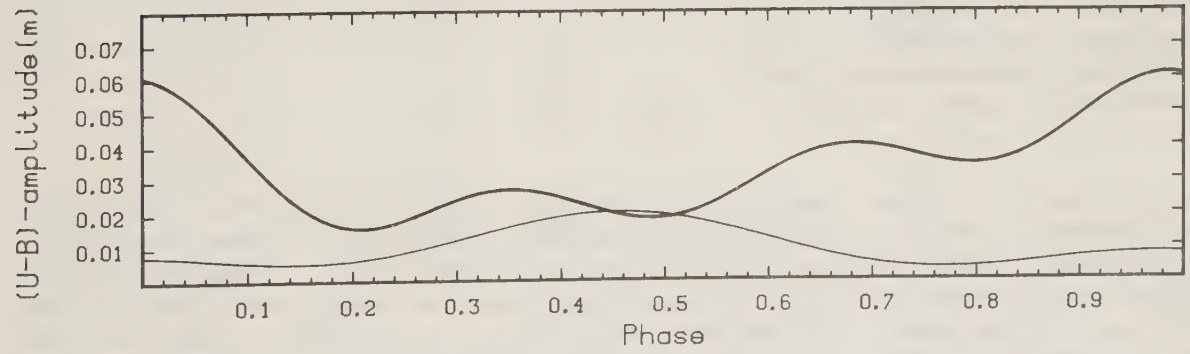

Fig. 3c

Table 17

A(m)	B(m)	N
6.160415	0.000000	0
0.007720	-0.002683	1
0.004238	-0.001591	2
0.003272	0.000205	3
0.000370	0.001962	4

In Fig.3 all the V ,B-V and U-B amplitudes and in Fig.4 all the corresponding mean curves are presented. The analysis includes only cosx and cos2x to describe the light curve and the thick curve corresponds to cosx and the thin curve to cos2x. There exist no significant differences between Fig.2a and 3a and Fig.2b and 4a which means that the introduction of cos3x in Fig.2a and 2b does not influence the cosx and cos2x curves. The Fourier coefficients for the V amplitude variation are given in Table 18.

Table 18

A(m)	B(m)	N	Curve
0.079363	0.000000	0	cosx
0.051873	-0.022900	1	cosx
0.017363	0.003268	2	cosx
0.010613	0.001657	3	cosx
0.000785	0.000678	4	cosx
0.011295	0.000000	0	cos2x
0.010293	-0.002378	1	cos2x
0.002224	0.000292	2	cos2x

The Fourier coefficients for the V mean shown in Fig.4a are given in Table 19.

Table 19

A(m)	B(m)	N
6.160037	0.000000	0
0.007358	-0.002782	1
0.003568	-0.001482	2
0.003918	-0.000771	3
-0.000015	0.001266	4

The B-V and U-B cosx amplitude curves (Fig.3b and 3c) have the same general shapes as the V amplitude curve and the epochs of maximum amplitude do not differ more than 1-2%. The variation of the amplitude is only 0.01mag greater for B-V than for U-B at maximum. The cos2x curve for B-V has two peaks. The highest peak is very close to the maximum for cosx which is expected but the other peak seems to be a compensation for a minimum in the cosx at the same phase. This part of the curves is not covered

with as many observatons as other parts. The Fourier coefficients for the B-V amplitude variation are given in Table 20.

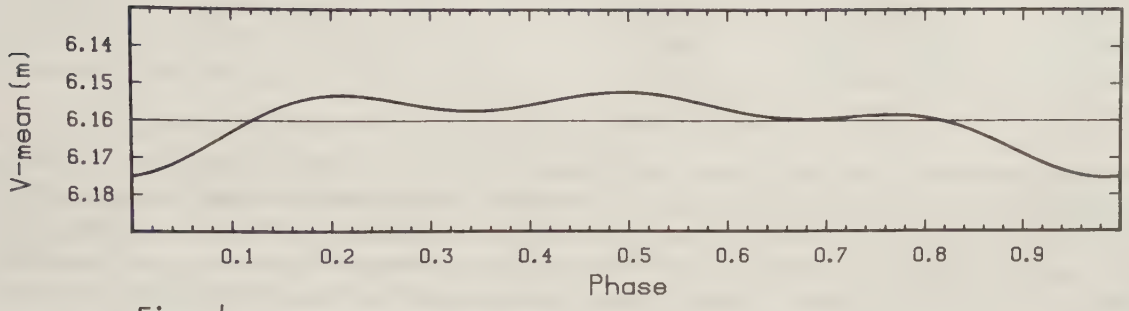

Fig. 4a

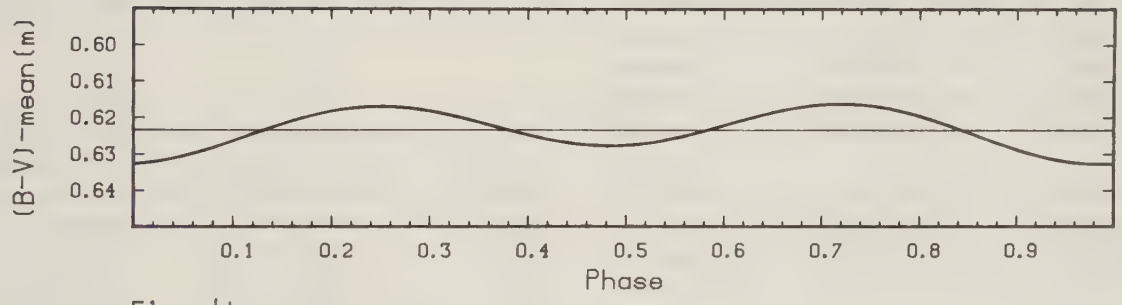

Fig. 4b

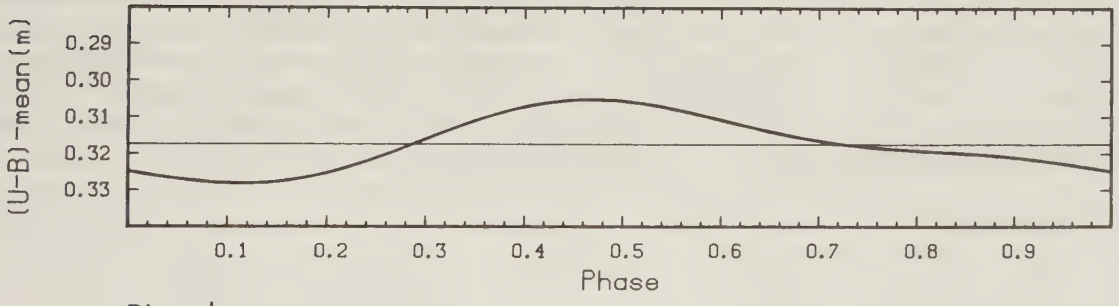

Fig. 4c

Table 20

A(m)	B(m)	N	Curve
0.037830	0.000000	0	cosx
0.025354	-0.009521	1	cosx
0.006021	-0.000720	2	cosx
0.003866	0.004283	3	cosx
0.006021	0.000000	0	cos2x
0.001460	0.000968	1	cos2x
0.003073	0.002694	2	cos2x

The cos2x curve for U-B has a clearly significant maximum close to the minimum of all the other curves and even worse the amplitude is comparable with the cosx amplitude. This makes the light curve very assymetric between phase 0.3-0.6. Several different analysis have been made with different combinations of orders of the Fourier series but this strange feature in the cos2x curve for U-B persists. The Fourier coefficients for the U-B amplitude variation are given in Table 21.

Table 21

A(m)	B(m)	N	Curve
0.033627	0.000000	0	cosx
0.011846	-0.009299	1	cosx
0.006069	-0.001651	2	cosx
0.003353	-0.000572	3	cosx
0.009392	0.000000	0	cos2x
-0.005029	0.002632	1	cos2x
0.003636	-0.002223	2	cos2x

The B-V and U-B observations have been analysed with third order Fourier series for cosx and second order for cos2x and the mean colour. The mean B-V curve is very similar to the V mean curve but the amplitude is smaller and the minimum occurs at the same phase (0.0) when the amplitude is at maximum. All features except the minimum are insignificant. The B-V mean curve can be found in Fig.4b and the corresponding Fourier coefficients are given in Table 22.

Table 22

A(m)	B(m)	N
0.623423	0.000000	0
0.002546	0.000027	1
0.006550	-0.001812	2

The U-B mean curve has an amplitude that is comparable with the V mean variation but the minimum occurs at phase 0.1 after the maximum amplitude. The shape is like a cosine curve. The mean

value of U-B for a complete Cepheid cycle is 0.023mag brighter when the amplitude is small than it is at maximum amplitude. The U-B mean curve is shown in Fig.4c and the corresponding Fourier coefficients are given in Table 23.

Table 23

A(m)	B(m)	N
0.317409	0.000000	0
0.009702	0.001143	1
-0.002008	0.002936	2

The variation of the radial velocity amplitude is demonstrated in Fig.5a. The cosx amplitude curve seems to be very nice and simple compared to the corresponding curve for the V-amplitude. The cosx and cos2x curves have their maxima at the same phase and take place about 4% earlier than the V maximum. The cos2x curve drops from 1.5km/s at maximum to less than 0.1km/s at minimum between the phases 0.3-0.4. The radial velocity amplitude is decreasing between phase 0.0-0.3 and is increasing first slowly and then more steeply from 0.3-1.0. The shape of the light curve varies from a perfect cosine curve at minimum to a 12% contribution from the cos2x at at maximum amplitude. The cosx curve is a fourth order and the cos2x curve is a second order Fourier series and the coefficients can be found in Table 24.

Table 24

A(km/s)	B(km/s)	N	Curve
5.775010	0.000000	0	cosx
4.182620	-2.279330	1	cosx
1.577880	-0.376450	2	cosx
0.479770	0.062430	3	cosx
0.057810	0.025330	4	cosx
0.689760	0.000000	0	cos2x
0.575000	-0.381330	1	cos2x
0.165810	-0.037630	2	cos2x

The mean radial velocity is discussed below and is plotted in Fig.3b.

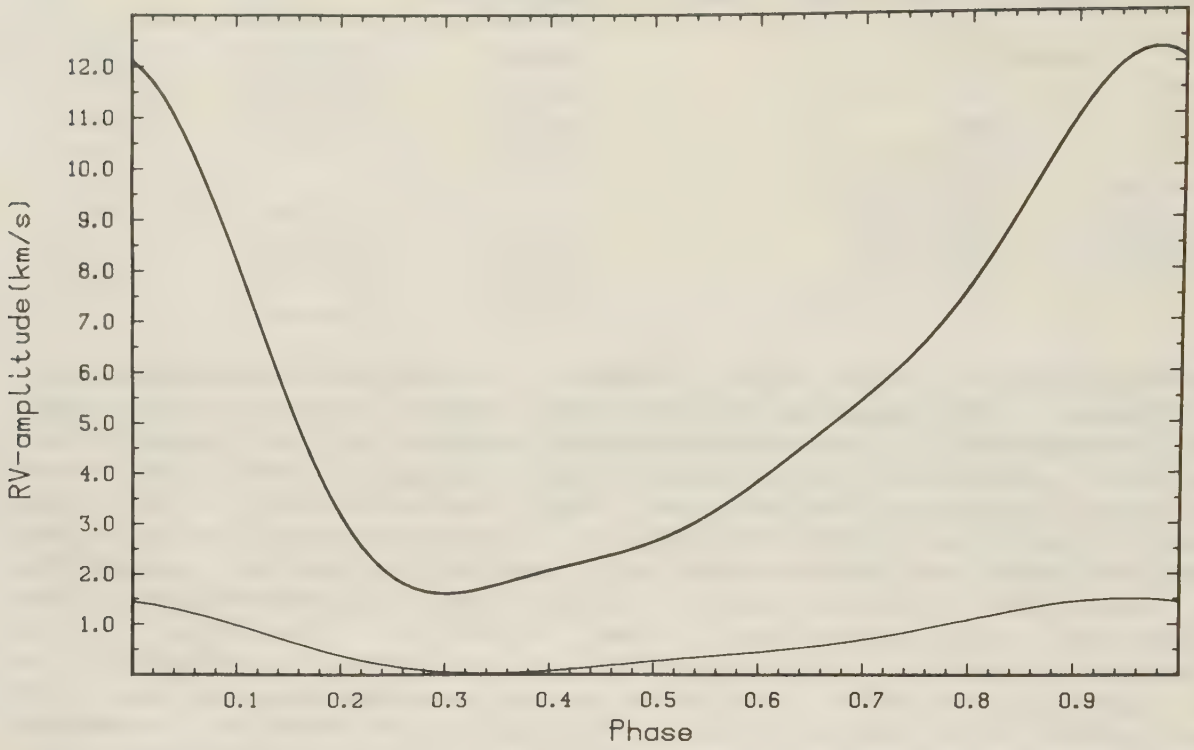
RV-amplitude(km/s)
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Phase

Fig. 5a

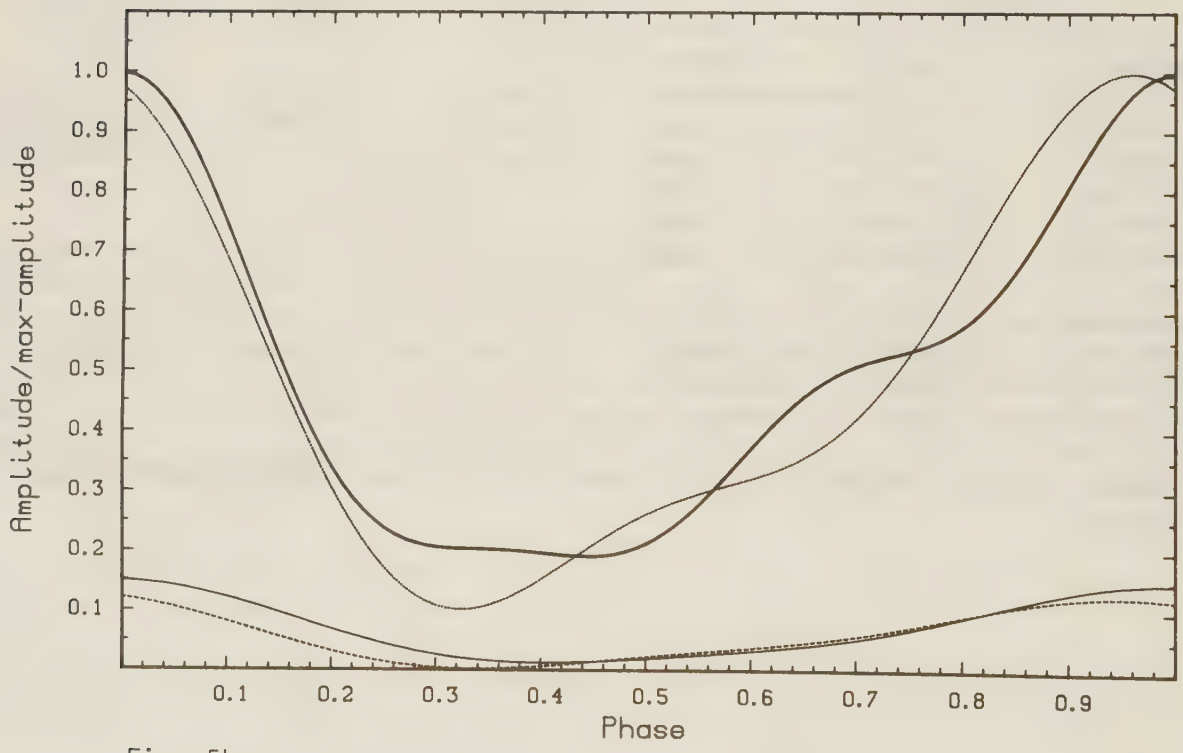
Amplitude/max-amplitude
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Phase

Fig. 5b

A direct comparison between the variation of the photometric and radial velocity amplitude can be made in Fig.5b, where the photometric and radial velocity curves have been divided by the corresponding maximum values of the cosx curves. The maximimum occurs 4% earlier for the radial velocity than for the V magnitude and it seems as the same thing is true for the whole curve except for the uncertain bending parts at phase 0.45-0.75. The same phase-shift can also be seen between the cos2x curves. The thick and thin solid curves are cosx and cos2x for the photometry and the dotted and dashed curves are cosx and cos2x for the radial velocity respectively. The explaination for the differences in shape and phase may be as is pointed out above that the radial velocity amplitude curve is based primarily on observations during one cycle and the more bending photometric amplitude curve is the result of superposition of observations from 5-6 cycles which are uncompletely covered. The four curves in Fig.6 describe the differences between the observed and computed time of the pulsational maximum and minimum for the Cepheid. (The zero-points are arbitrary). The thick solid curve represents the V magnitude, the thin solid curve the radial velocity, the dotted curve the B-V colour and the dashed curve the U-B colour. The computed epoch means here eqiudistant pulses and constant period. It became clear early during the study of HR7308 that the period was not constant througout the amplitude variation cycle. The explaination to the curious effect that the radial velocity period seems to be more variable than the V magnitude period may again be that the variation of the radial velocity period is determined during one cycle while the V magnitude period variation is determined from 5-6 different cycles of amplitude variation. For all the parameters the periods are getting shorter between the phases 0.95-0.20, which corresponds to the maximum and the decreasing part of the amplitude cycle. The period is longer than the mean value from phase 0.2 to about 0.4, corresponding to the minimum amplitude part with nearly constant amplitude.

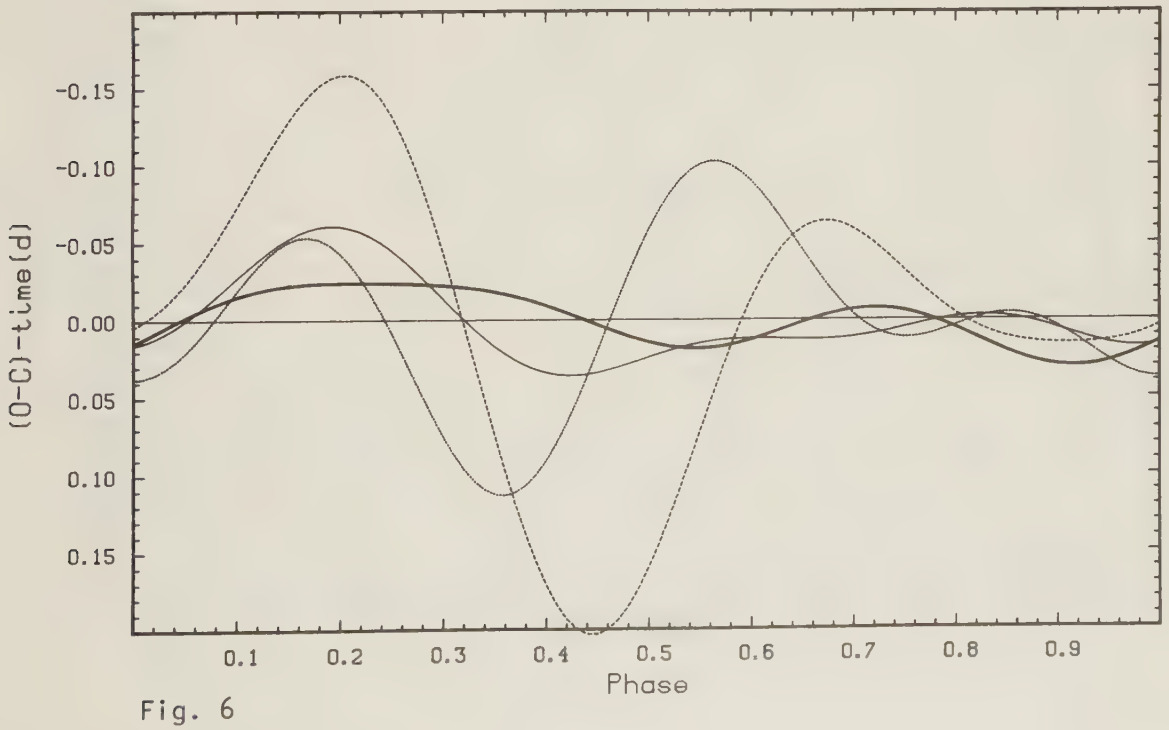

Fig. 6

The period is quite constant for the radial velocity and the V magnitude from phase 0.5 to 0.95 when the amplitude is clearly increasing. For the B-V and U-B colours the period is getting shorter between the phases 0.4-0.6 and are then getting longer until phase 0.95. This means that when the amplitude changes from being constant or only slowly increasing to an obvious increase this does not effect the radial velocity and the V period as much as the B-V and U-B periods. The time-shift that is illustrated in Fig.7a was much more difficult to discover than that shown in Fig.6. The period was first thought to be 5*Pa and modified to 2.5*Pa but it has now been determined to be 2.25*Pa=2655days(see above). The phase 0.0 corresponds to J.D. 2443597.7 as for the other diagrams, and the 'Phase' is computed from Pa=1178.5days. The curve is determined only from the photometry and from observations made during 1962-1982. The fact that the period Pt is a rational multiple of Pa must be important for the understanding of HR7308. The period is longer than the mean value during 75% of Pt and shorter during only 25%. There are reasons to expect that the shape of the amplitude variation curve shown in Fig.2a and 2b is the result of superposition of cycles with different shape and length and therefore an investigation of a possible correlation of the mean-amplitude(constant term in the Fourier series in Table 16) with the period Pt=2655days. The variation of the mean-amplitude of the V magnitude is shown in Fig.7b. From this diagram it is evident that strange bends on the V-amplitude curve can be explained by differences between the amplitude cycles even if it is not obvious that Pt is the best period for this variation (if periodic at all). The effect of the variation of the amplitude cycles can be seen in Fig.7c.

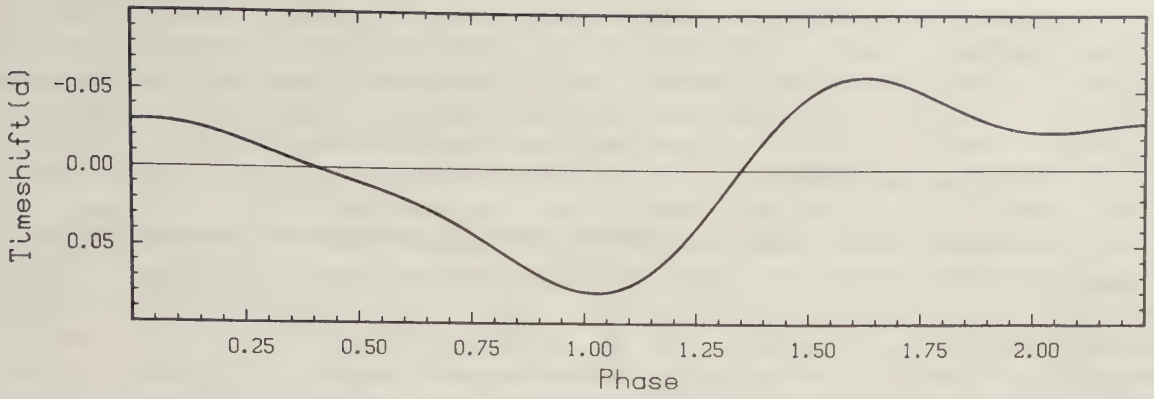

Fig. 7a

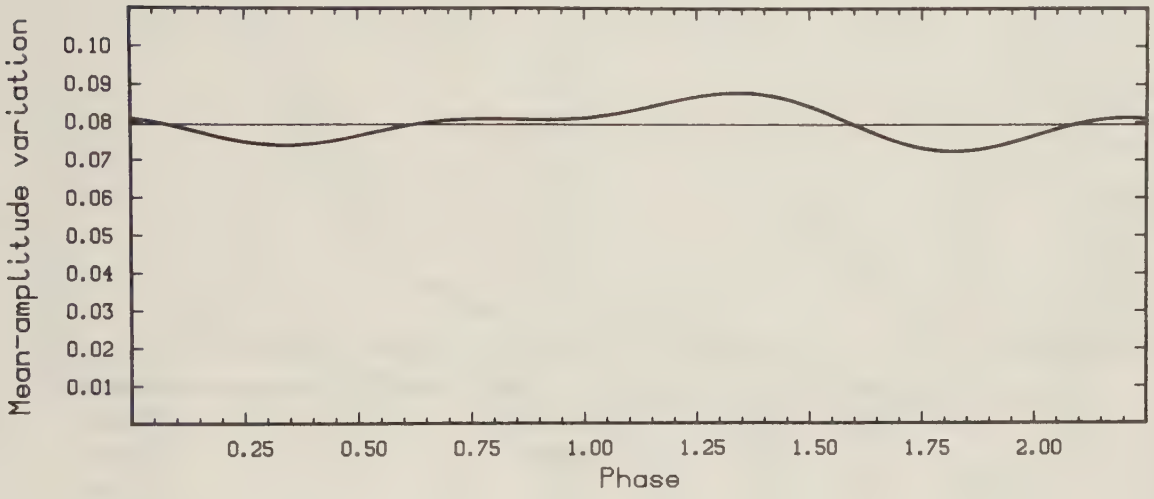

Fig. 7b

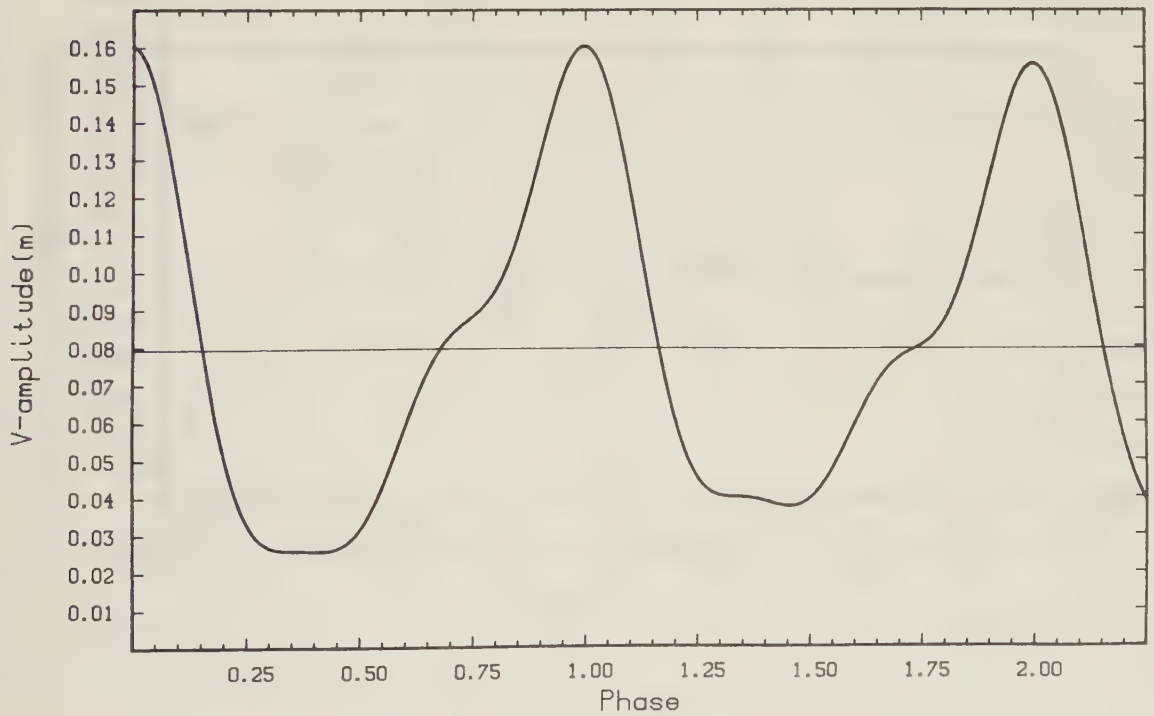

Fig. 7c

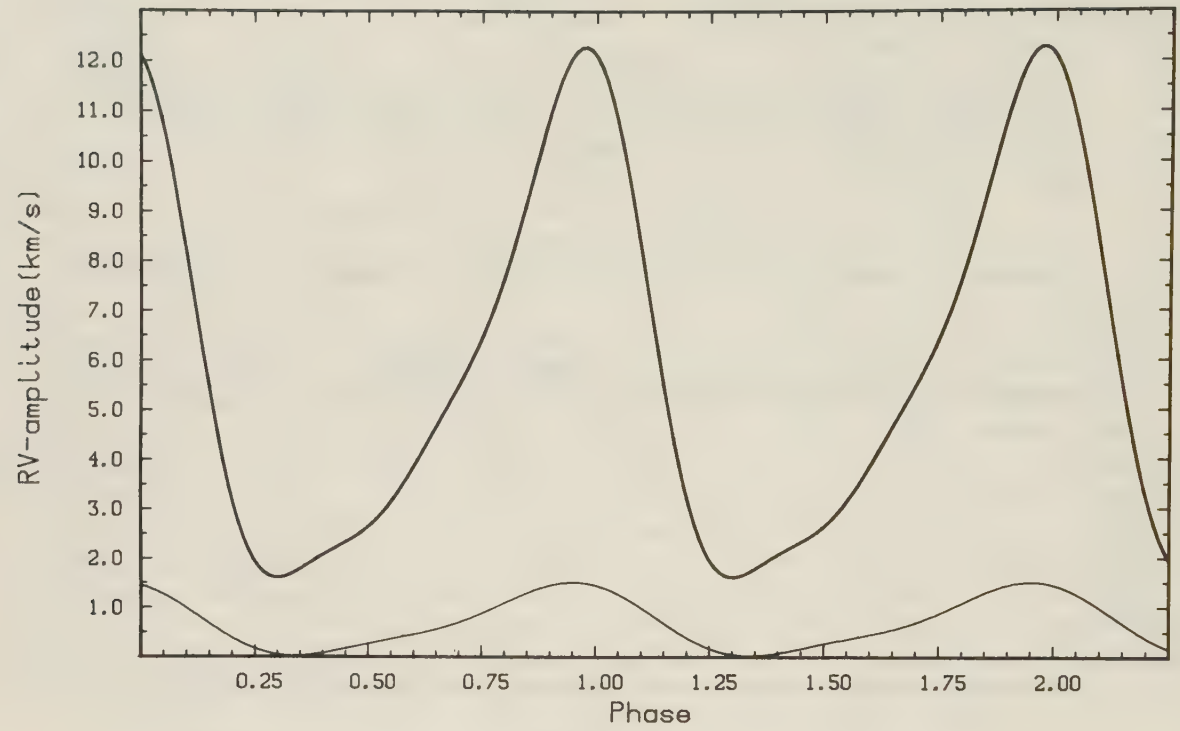
RV-amplitude(km/s)
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Phase

Fig. 8a

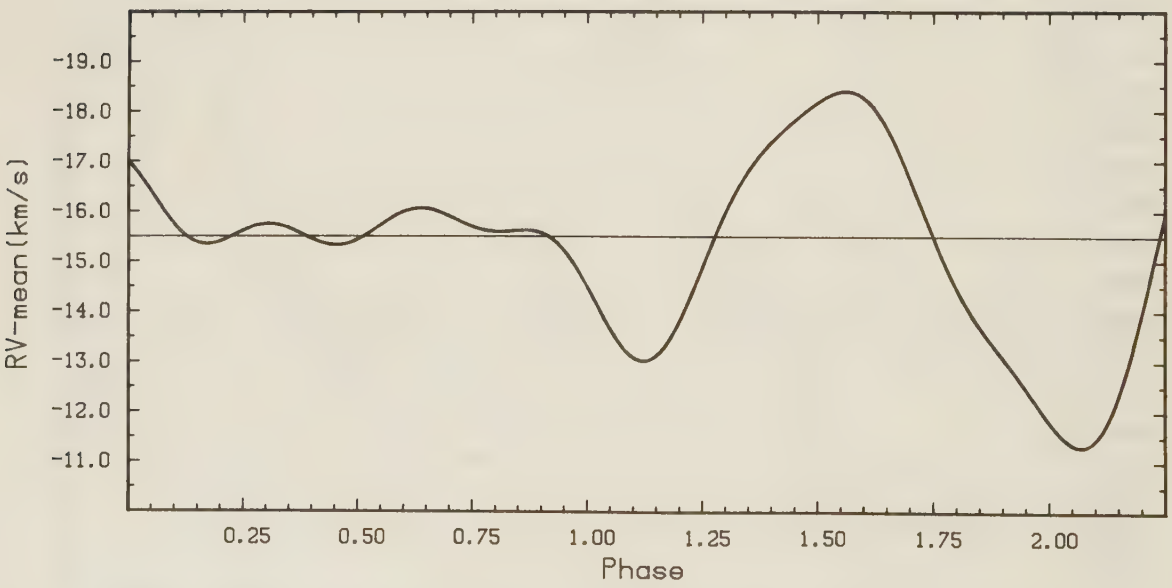
RV-mean(km/s)
-19.0
-18.0
-17.0
-16.0
-15.0
-14.0
-13.0
-12.0
-11.0
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Phase

Fig. 8b

In Fig.8a and 8b the radial velocity amplitude and mean-velocity of the Cepheid during the pulsational cycles are plotted. The phase interval is the same as in Fig.7. The radial velocity mean-curve is very uncertain for phases greater than 1.0 because no observations are made outside the interval -0.05 to 1.0 except for the observations at phase 5.5 by Albitzky and Shajn 1929-1930. The curve is the sum of two Fourier series one with the period Pa=1178.5days and the other with the period P4=1063.5days. The curve in Fig.9 shows the behaviour of the radial velocity mean curve model during nine Pa cycles which is equal to ten P4 cycles. This curve gives of course only a qualitive idea of the true motion of the Cepheid because observations during 29 years are necessary to cover the whole interval. The interpretation of the motion of the Cepheid is made below.

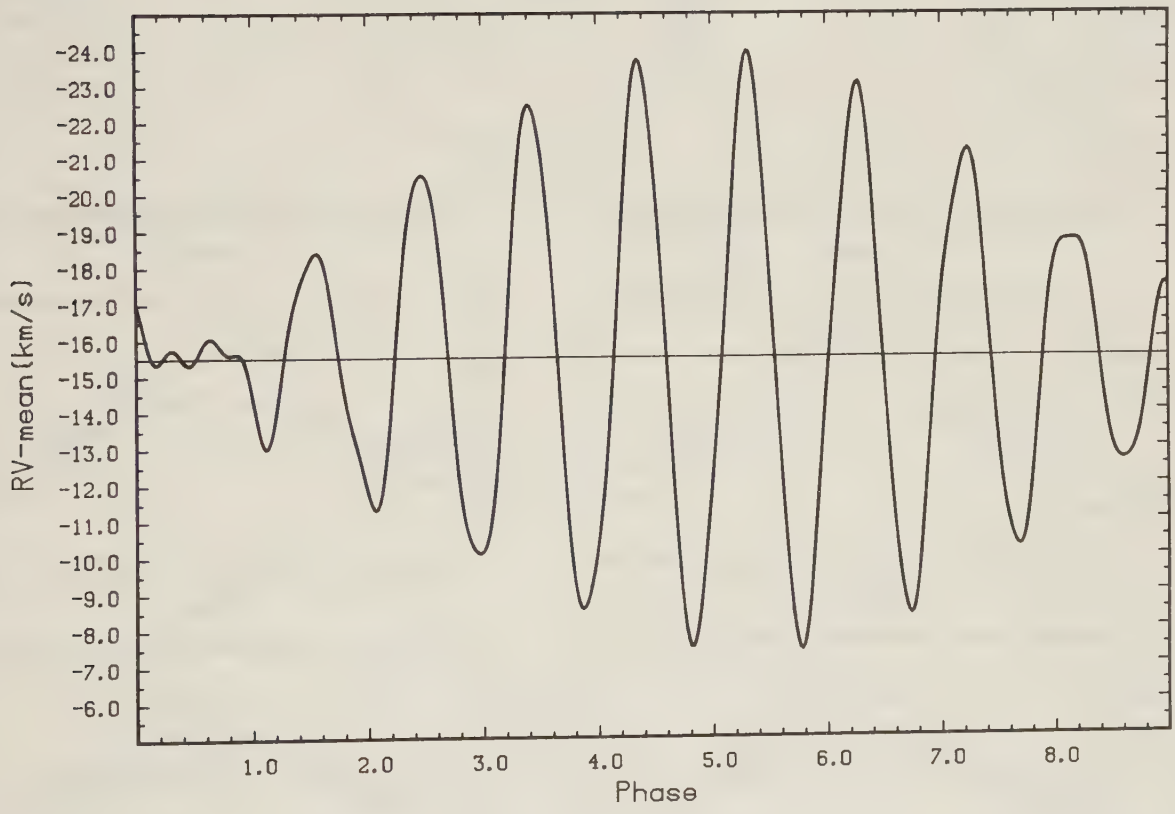

Fig. 9

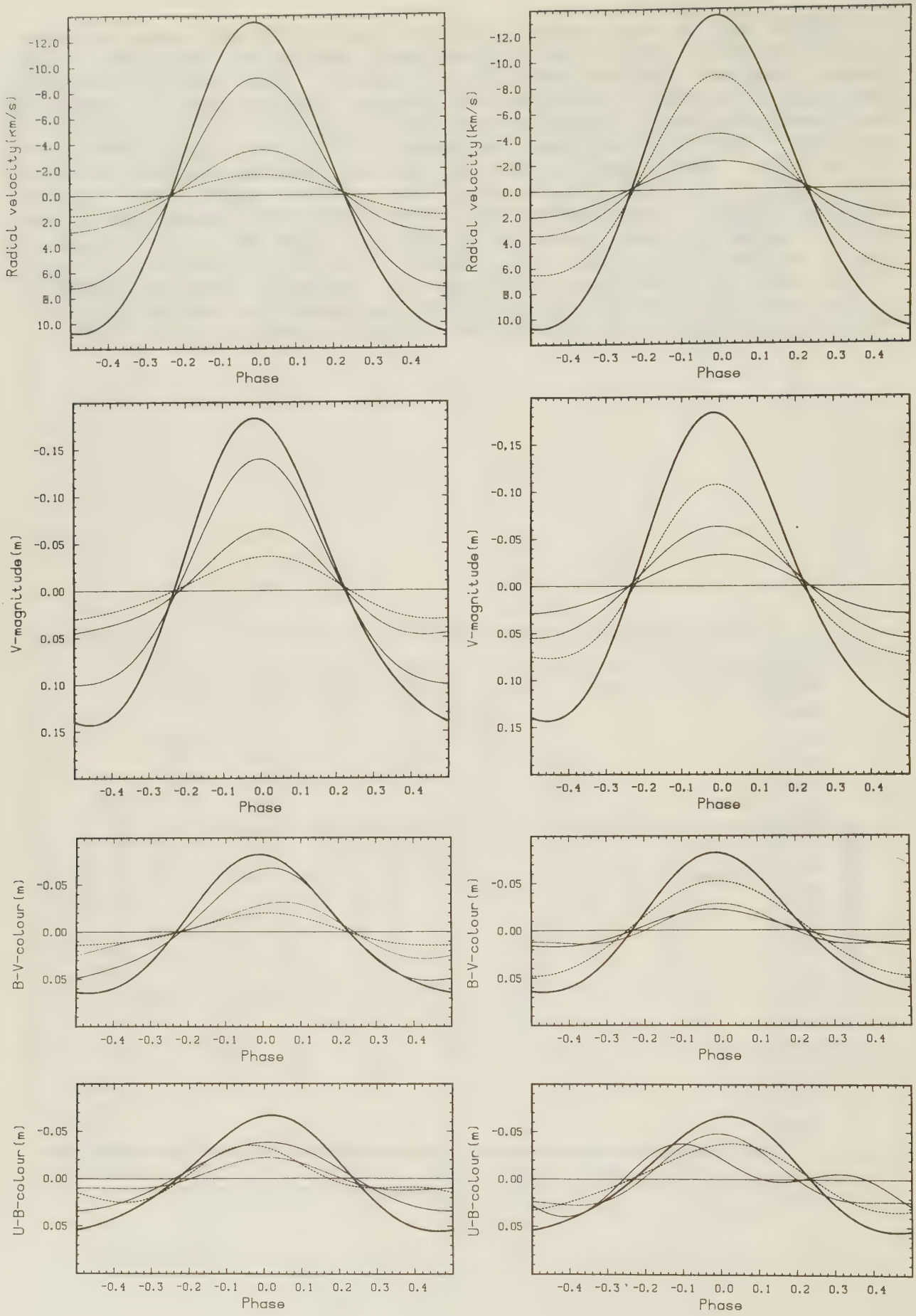
Radial velocity(km/s)
-12.0
-10.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0
10.0
-0.4 -0.3 -0.2 -0.1 0.0 0.1 0.2 0.3 0.4
Phase
V-magnitude(m)
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
B-V-colour(m)
U-B-colour(m)

Fig. 10

In Fig.10 we can see the V,B-V and U-B pulsational curves. The thick solid curves represent the light curve at amplitude phase 0.0(maximum amplitude) in all the diagrams. For the curves to the left the thin solid line, the dotted line and the dashed line represent the amplitude phases 0.1,0.2 and 0.3 respectively. For the curves to the right the corresponding amplitude phases are 0.4,0.6 and 0.8.

AN ATTEMPT TO MAKE A DYNAMICAL MODEL FROM MOTIONS OF THE CEPHEID

The author has since 1976 tried to explain the variation of the amplitude of the light curve of HR7308 as the result of disturbances from a companion However no traces of a secondary has so-far been discovered in the spectrum from the blue region or from the ultra-violet IUE spectra(see Fig.11). If the secondary star has a mass that is not small compared to the Cepheid and the orbit is not too close to the celestial plane the existence of the secondary can be verified from the variation of the mean velocity of the Cepheid. At the meeting in Liege the author expected this variation to be about 10km/s(if the curve is sinusoidal) from the old observations 1929-1930 and the mean value published from DDO 1934-1938. When it became possible to use the modern observations 1977-1982 it was clear that for these data the variation of the mean velocity of the Cepheid during an amplitude cycle is less than 1km/s. There seems to exist a mechanism that changes the amplitude of mean velocity. The author suggests that this phenomenon can be explained if the orbital plane of the secondary is precessing in such a way that the inclination of the orbit to the celestial plane is much smaller now than it was 1929-1930. The orbit of the Cepheid is not a closed ellips and the apsidal and nodal lines are precessing. If Pa=1178.5d is the synodical period and the siderial period is P4=1063.5d we can explain the necessity to introduce this fourth period. However,if there does not exist a third body in this system the excentricity of the orbit of the secondary must be about 0.95 which means that the two stars touch each other at periastron. This problem has been studied with the theory given by Kopal(1959) for perturbations in binary systems. If we accept the existence of a third star we can explain the apparent anomalies in the observed velocities and even understand the 2655d period. The connection between the different periods is given in Table 25.

Table 25

Phenomenon	Period (day)	N (10621/P)	Pobs
Rotation of the line of apsis in the inner system	10621.00	1	
Rotation of the line os nodes in the inner system	7080.67	3/2	
The three stars lined up	2655.25	4	2655.0
Siderial period of the outer star	1327.63	8	
Synodical period of the inner star=Pa	1180.11	9	1178.5
Siderial period of the inner star	1062.10	10	1063.5

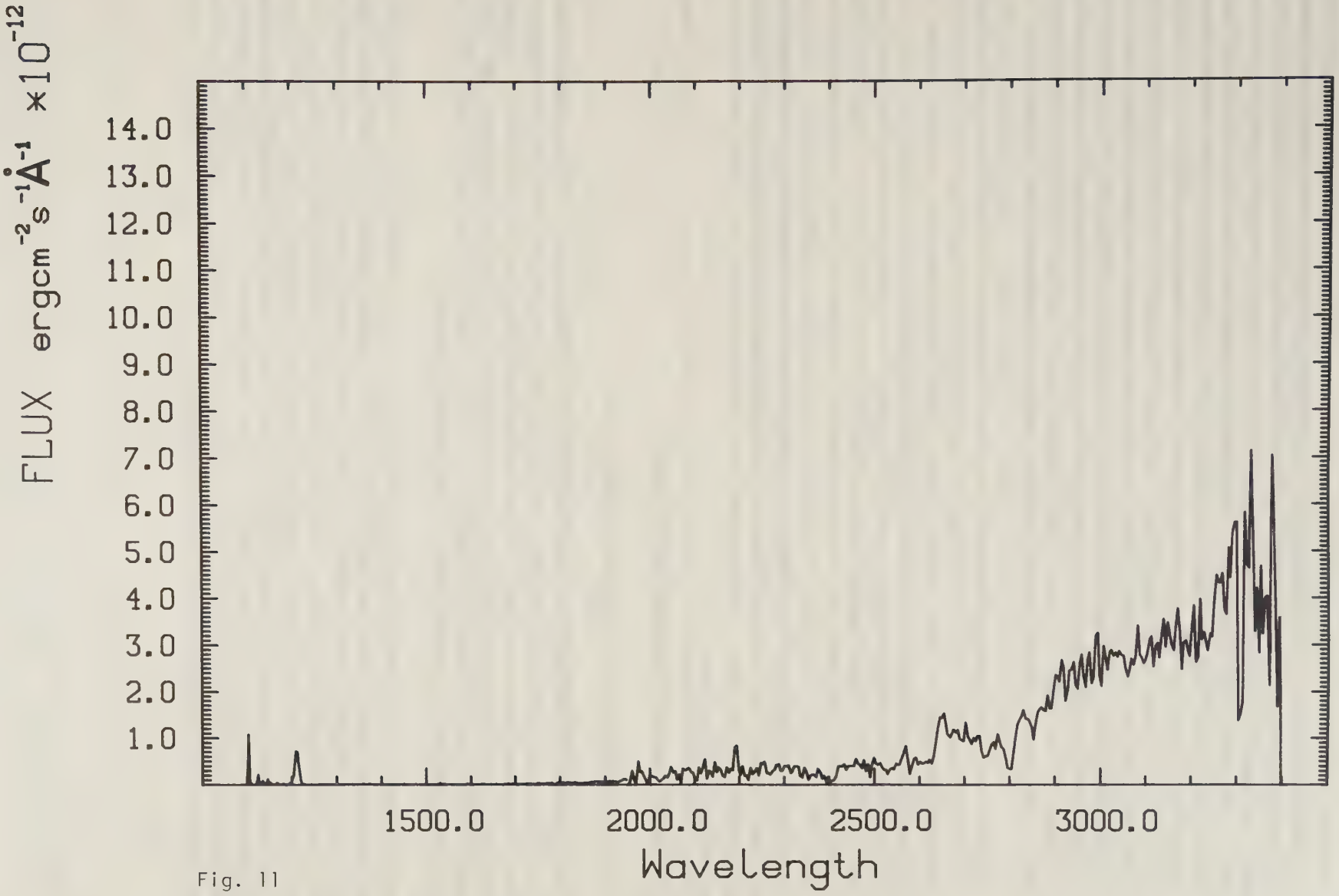
FLUX ergcm^{-2}s^{-1}Å^{-1} *10^{-12}
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
1500.0
2000.0
2500.0
3000.0
Wavelength

Fig. 11

In order to investigate if there exists a stable three body system with the above specified properties the author used formulas from Kopal(1959) originally derived by Delauney. A new computer program was made that computes the perturbations of the inclination, excentricity, line of nodes and line of apsis for the component closest to the most massive component, the Cepheid. The outer component is moving in an elliptical orbit with fix orientation in space.The perturbations for the inner orbit have been computed for every second degree of the true anomaly of the outer star and have been applied to the elements of the inner orbit before the next computation. A great number of combinations of inclinations and excentricities for the two orbits and mass of the outer component relative to the total mass of the system have been tested. For the properties specified in Table 25 there seems to exist a unique solution presented in Table 26.

Table 26

k=mC/(mA+mB+mC)	0.286
Inclination between the orbits	23.5degrees
Excentricity in the inner orbit	0.28
Excentricity in the outer orbit	0.10

If the amplitude of the mean velocity of the Cepheid, 0.3km/s in 1981 and 5km/s in 1930, are the minimum and maximum amplitudes respectively we can determine the inclination of the orbits to the celestial plane:

sini(1981)/sini(1930)=0.30/5=0.06

i(1930)-i(1981)=23.5degrees(Table 26)

which gives i(1981)=1.4 and i(1930)=24.9degrees. From the theory of stellar evolution,the mass of the Cepheid can be estimated to be 5.2Msun(see below). The observations by IUE give an upper limit for the masses of the components <2.5Msun because no flux from a hot star can be found. The mass ratio in Table 26 is e.g. satisfied by mB=0.8Msun and mC=2.4Msun.

These simple calculations show that it is possible to explain the periods and the variations in the mean velocity of the Cepheid by a three body model. The inner component is responsable for the variation in the pulsational amplitude while the variation of the pulsational period has two periods one correlated with the distance to the inner star and one correlated with the alignement of the three stars. The variation of the pulsational amplitude would thus the caused by gravitational disturbance in the pulsating envelope of the Cepheid.

DETERMINATION OF THE RADIUS, NONRADIAL OSCILLATIONS

In this chapter the radius of the Cepheid is determined by a formalism developed by Dziembowski(1977),modified by Balona and Stobie(1979a,b) and by the Wesselink method Wesselink(1969) combined with the Barnes-Evans relation Barnes,Evans(1976), and the principles used for Cepheids discribed in Barnes,Evans and Parsons(1976) and in Barnes,Dominy,Evans,Kelton,Parsons and Stover(1977). The unusual variation of the amplitude of the Cepheid HR7308 makes it natural to extend the explainations not only to radial but also to nonradial oscillations. Dziembowski showes that it is possible to determine the oscillation mode by application of the Wesselink technique. The radius determined in this way is sensitive to the spherical harmonic order of the oscillation. Balona and Stobie have formulated the principles of Dziembowski in a way that is more practically useful. Instead of the effective temperature concept they use the surface brightness S=-2.5logF where F=L/A means the observed luminosity per unit area. Wesselink(1969) has shown that S is correlated with (B-V)0 and Barnes and Evans(1976) demonstrates that this correlation is even better for (V-R)0. The advantage with the formulations by Balona and Stobie is that they involve only quantities that are directly observed V,B-V, radial velocity RV and the corresponding phase lag relative the radius-variation. The disadvantage is that the method breakes down for odd values of l. The general formulas given by Balona and Stobie are:

$$\frac{DL}{L} = e^{*}P_{lm}(my_0)\,(f_1^2 b_1^2 + g_1^2 + 2 f_1 b_1 g_1 \cos(psi))^{1/2} \cos(omega\,t + m\,fi_0 + alfa) \tag{24}$$

$$\frac{DF}{F} = e^{*}P_{lm}(my_0)\,(f_1^2 b_1^2 + h_1^2 + 2 f_1 b_1 h_1 \cos(psi))^{1/2} \cos(omega\,t + m\,fi_0 + beta) \tag{25}$$

$$DV_r = e^{*}P_{lm}(my_0)\,omega\,R_0\,(u_1 + alpha\,v_1)\cos(omega\,t + m\,fi_0 - pi/2) \tag{26}$$

$$\tan(alpha) = f_1 b_1 \sin(psi)/(g_1 + f_1 b_1 \cos(psi)) \tag{27}$$

$$\tan(beta) = f_1 b_1 \sin(psi)/(h_1 + f_1 b_1 \cos(psi)) \tag{28}$$

where

$$g_1 = 2 b_1 - c_1 \quad , \quad h_1 = 2 b_1 - c_1 - 2(2 p_1 - q_1) \quad , \quad alpha = G M/(omega_H^2 R_0^3) \tag{29}$$

G is the gravitational constant, M the mass of the star, omega the frequency of the pulsation, $my_0=\cos(Teta_0)$ where Teta0 is the aspect angle and Plm(my) is the Legendre function.

The remaining constants ul,vl,bl,cl,pl and ql are integrals that involve the Legendre polynomials Pl = P(my) and the limb darkening law h=h(my). The Eddington limb darkening law h=1 + 1.5*my has been used in the numerical computations but the author of this paper has not used alphaH=0.08 as Dziembowski and Balona and Stobie(more useful for Delta Scuti stars) but computed it from the formula. The relations can in principal be used to determine all the unknowns except the azimuthal parameter m. For all odd values of l the equations are reduced to the special case hl=gl, alpha=beta and DL/L=DF/F. The luminocity variations are directely determined by the flux variations and this occurs because for odd values of l the projected area variation is zero which makes the Wesselink technique impossible.

According to Balona and Stobie(1979b) it can be shown that if the light, colour and velocity observations are decomposed into sinusoidal components the system of equations holds for each harmonic component provided that the amplitude is not too large. This technique has been used by the author to determine the radius of HR7308 from the first order harmonic component.

If one knows the radius from other methods it is possible to determine the value of l the number of nodes on the surface of the star. This is possible due to the fact that the observed radial velocity of a pulsating star is not identical with the true motion of the surface along the line of sight but is a mean of the motions of the limbdarkened surface of the visible hemisphere. To determine the radius of a pulsating star using the above mentioned methods one has first to integrate the radial velocity curve to get the variation of the radius due to the pulsation. To get the real variation of the radius the integrated value must be multiplied by a factor p that depends on the value of l. For the radial modes(l=0),this factor is 24/17=1.412 from the classical theory by Carroll(1928) and Getting(1935). The real line profile is assymmetrical and therefore the measured value depends on the line width or resolution and on the method of measurement. Parsons(1972) has used model atmospheres to study these effects and he has found that for most variable stars p=1.31 for plate dispersion about 10Å/mm(and little macroscopic broadening) and p=1.34 for plate dispersions greater than 40Å/mm. The most appropriate values for classical Cepheids is p=1.31 and this value has been used in this paper. For the nonradial oscillations this factor depends strongly on l if l is greater than three but also on the mass,radius and period of pulsation. If for example the mass is 5Msun,the radius is 34Rsun and the period is 1.5days (reasonable values for HR7308),the factor is 1.31 for l=0, -1.375 for l=2 and 13.05 for l=4. If the Wesselink method is used it is very easy to chose between l=0 and l=2 because if the true l is equal 2 the radius will become negative. If on the other hand the true value of l is 4 the computed value of the radius will be about ten times smaller than expected for l=0. For the true value of l equal 1,3,5 or other odd numbers the formulas given by Balona and Stobie are not defined and will give useless values of the radius because alpha=beta=psi which means that f becomes infinitely large. The most independent method to determine the radius is the application of Barnes-Evans relation to Cepheids. The integrated velocity curve with an amplitude multiplied by a correct factor gives a value of the variation of the radius in linear

measure and this is compared with the amplitude of the corresponding variation of the angular diameter(in milliarcseconds) given as:

$$\log(fi)=0.5134-0.2*(V-A_V)+2*C*((B-V)-E_{B-V}) \quad (30a)$$

The value of C must be chosen correctly and depends if B-V are used as the observed colour both on spectral class and luminosity. Thompson(1975) gives C=0.211 for Cepheids and other observers gives also values close to 0.20. Bell and Gustavsson(1980) gives C=0.265 for G giants and Barnes, Evans and Parsons(1976) gives C=0.333 for a wide range in spectral type and mostly dwarfs. The value C=0.20 gives the best agreement between the different methods for the determination of the radius of HR7308.

I have found that for Cepheids the radius and distance computed from (30a) and (31) deviate from the Stephan-Boltzmann law with more than 30%. However,if we use the original equation by Wesselink(1969) and use the log Te computed from the Kraft relation and corrected for metal content from Iben and Tuggle(1975)(equation (49) below) and the bolometric correction from Sandage and Tammann(1969) (equation(50) below),we will get a very usefull relation:

$$\log(fi)=0.6404-0.2*(V-A_V)+2*0.2072*((B-V)-E_{B-V})-2.66*(Z-0.02) \quad (30b)$$

The value of the constant C=0.2072 . In the computations the values of E(B-V)=0.07 and R=3.3 have been used.

A least square fit of dfi computed from the observations of V and B-V and dR from the integrated velocity curve is made to get the distance d from:

$$0.5*dfi=a+d*dR \quad (31)$$

where a and d are constants that should be determined and dfi and dR are the deviations from the mean value of fi and R respectively, caused by the pulsation. An important parameter is the phasing of the variations of fi and R. The phase lag is optimized from different solutions to give minimum errors in dfi. When we know the distance and the mean angular diameter it is straight-forward to get the mean-radius. It was at first a very great surprise to the author that the phase difference between maximum radius along the line of sight and the maximum angular diameter is almost 180degrees. Therefore it was very interesting to read the papers by Dziembowski and Balona and Stobie that can explain this effect by nonradial oscillations with l=2.

The Wesselink radius is determined by the formulation given by Balona(1977) but with a least square fit to the equation with the data taken from the Fourier curves instead of a determination of the unknowns by the principle of maximum likelyhood from the observations and their estimated errors. The relation is:

$$V = \text{constant} + C*(B-V) + 5*\log e*dR/R_0 \quad (32)$$

The parameters that should be determined are the constant,C(defined above) and the mean-radius R0. In all the computations with the methods by Barnes and Evans and by Wesselink the observed quantities V,B-V and radial velocity are taken from the Fourier model with first and second order terms of the pulsation curves. The observations from which this Fourier model is computed is limited to the interval 1972-1982 to get a sufficient number of observations and as simultanous as possible.

The different determinations of the radius are presented in Table 27 together with the constant C in the Wesselink method. The 'variation' of the radius with Pa phase is very clear.

Table 27

Pa-Phase	10*C (Wesselink)	R(Rsun) (Wesselink) l=0	R(Rsun) (Barnes-Evans)* l=0	R(Rsun) (Balona-Stobie) l=0	l=2	l=4
0.00	2.025	31.73	32.30	33.1	34.8	329
0.10	2.035	28.15	30.01	29.6	31.0	294
0.20	1.986	14.92	15.49	13.9	14.6	138
0.30	1.892	9.59	11.69	25.5	26.7	253
0.40	0.462	5.78	4.92	9.2	9.7	92
0.45	1.034	6.06	5.08	5.7	6.0	57
0.50	1.814	5.37	5.20	0.8	0.9	8
0.55	1.596	8.32	8.57	4.8	5.1	48
0.60	1.950	13.22	15.15	18.3	19.2	182
0.70	2.107	16.86	17.08	14.2	14.9	141
0.80	2.141	22.45	22.99	24.1	25.3	239
0.90	2.065	32.02	33.65	32.2	33.8	320

*The constants derived by Henriksson has been used(30b).

The values computed with the Barnes-Evans method gives a smoothly varying radius with the same shape as the variations of the amplitudes of the photometry and velocities. At maximum near phase zero the radius is 32Rsun and at minimum near phase 0.5 it is 5?sun. The p-factor used for Barnes-Evans and Wesselink corresponds to l=0 and the true value of the radius if l=2 is found if these values are multiplied by 1.375/1.31=1.050 and for l=4 by 13.05/1.31=9.962.

In Fig.12 the values of alpha(circles),beta(triangles) and psi(pluses) for l=2 can be seen. When alpha=beta we have an odd value of l and this explains the high values of R(Balona-Stobie) in Table 27,at phase 0.30 and phase 0.60. (Their theory is not valid if alpha is close to beta).

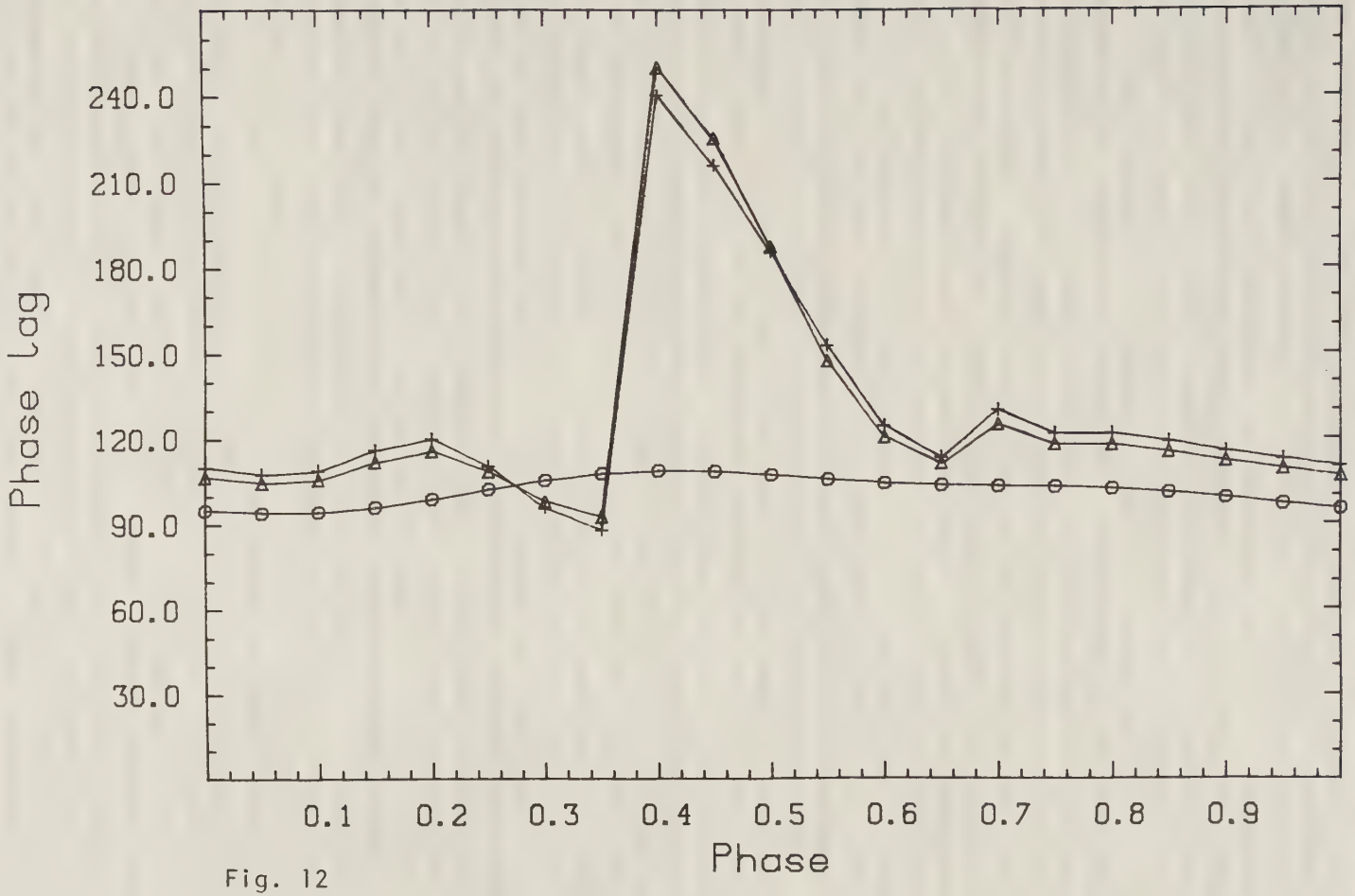
240.0
210.0
180.0
150.0
120.0
90.0
60.0
30.0
Phase Lag
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Phase

Fig. 12

It may be possible to determine the values of l and m from the variation of the amplitude in V,B-V and RV and the computed radius. The three equations that describe the variation of L,F and RV are all multiplied by the factor e*Plm(my0). It is not necessary to know this factor,for the solution of R0 and R0 is independent of e,m and the aspect angle Teta0(my0=cos(Teta0)). The index l of the Legendre function is involved in many important ways concerning the amplitudes but m is only involved through the phase. For a normal Cepheid with constant amplitude and nearly constant period it is impossible to determine the azimutal index m. If the amplitude is variable it is possible to determine the variation of l if we assume that the mean radius R0 is constant. It is therefore possible to compute the factor e*Plm(my0) from the three equations that describe the variation of the amplitudes. If we can assume that e and my0 are constant it is now possible to determine m and my0 from a tabulation of Plm(my0) as l is known. Unfortunately it seems not reasonable to assume that e is constant without knowledge of the mechanism that causes the variation of the amplitude. It seems less dangerous to assume that my0 is constant even if the Cepheid is suspected to be a member of a multiple system because the period of precession must be several hundreds of years for such a centrally condensed star. A variation of the period for a nonradially oscillating star may be caused by a change of m if also the amplitude is variable. It is possible to determine the value of l from the scatter in the three computed values of e*Plm(my0). If the assumed value of l is correct the scatter will be very small. The variation of this factor is for l=2 very similar to the variation of the amplitudes but as Plm(my0) is constant or changes abruptly for a sudden change of m we must have a slow variation of e if m is constant. The least scatter between the three determinations of the factor is found for l=2 at the phases of Pa equals 0.0-0.35 and 0.60-1.00 and for l=4 at 0.40-0.55. The maximum value of the factor is 0.0297 at phase 0.00 for l=2 and 0.1544 at phase 0.50 for l=4. If e is the same at these phases the value of Plm(my0) must be about 5.2 times greater at phase 0.5 compared to phase 0.0. The corresponding relation for the computed Plm(my0) values is for Tet0=15degrees 3.089 and for Tet0=20degrees 5.513. A possible solution is that Tet0=19.4degrees and that l=2 and m=0 at phase 0.00 and l=4 and m=2 at phase 0.50. A nearly pole-on solution fits very well the phase-shifts found in the Barnes-Evans method of determination of the radius. A change of m will also give a change of the period at least if the azimuthal component of the pulsation is the most important.

If l>0 we have Pobs = Pmode/m.

If m is changed from 0 to 2 this must mean that the observed period for HR7308 should be only 0.5*PC at Pa-phase 0.5 if the period is PC at phase 0.0. This it not observed for the V and RV-curves which means that the azimuthal component is much smaller than the latitude component and this is another argument favouring the nearly pole-on solution. However,if we look at the colour curves B-V and U-B we will find that the amplitude of 0.5*PC is comparatively stronger at Pa-phase 0.5 than at Pa-phase 0.0 especially for U-B where the amplitude of 0.5*PC and PC is equal. The effect of the variation of m is stronger at shorter wavelengths. Other combinations of Teta0, l and m seems less reasonable. If we assume that the Cepheid is a member of binary system the companion may at phase 0.0 be at periastron

and cancel the azimuthal components of the nonradial oscillation by tidal interaction. Numerical computations by Osaki(1977) show that a Cepheid model with l smaller than 4 is stable against nonradial oscillations at least within the linear stability analysis. (The model is pulsationally unstable against the lowest radial modes.) For l=4 only the first harmonic oscillation is unstable. The Cepheid in HR7308 may be oscillating in the first harmonic with l=4 and m=2 at the least disturbed position of the orbit at apastron around phase 0.50 and when the companion comes closer the equatorially oscillating azimuthal component becomes more and more damped but with the latitude component unchanged.

From Unni,Osaki,Ando and Shibahashi(1979) page 240 the damping time is defined as:

$$\tau_{damp} = -\frac{1}{\sigma_I} \qquad (33)$$

and on page 164 we find that if $|\sigma_I/\sigma_R| \ll 1$ the period $P = 2\pi/\sigma_R$.

The growth rate of the amplitude is on page 265 defined as:

$$\eta = \sigma_I/\sigma_R$$

If these formulas are combined we get:

$$\tau_{damp} = -\frac{P}{2\pi\eta} \qquad (34)$$

The numerical calculations have been performed for a Cepheid model having the same external parameters as those of Baker and Kippenhahn's (1965) model 7. They are

$M = 7M_{sun}$, $R = 80.3R_{sun}$, $\log T_{eff} = 3.726$, $X = 0.70$ and $Z = 0.03$.

Table 28 shows the result by Osaki for l=0 and l=4.

Table 28

l	Mode	Period (day)	eta	w leak /(4pi)
0	F	11.57	+0.0058	-
	1H	8.39	+0.0087	-
	2H	6.35	+0.0023	-
4	$\bar{f}$	11.49	-0.011	-0.014
	$\bar{p}_1$	8.04	+0.0016	-0.0059
	$\bar{p}_2$	5.93	-0.0030	-0.0032

If we compute the damping time we get -2.2years

and +0.86years for $\bar{p}_1$ and $\bar{p}_2$ respectively.

This means that the time for growth of the amplitude in the first case is about 2years and damping in the second case is about 1year. The time of growth and damping for this model is of the same order of magnitude as Pa =3.2years. This model may give an idea of the mechanism for the amplitude varation of HR7308 but a model with smaller mass and a radius equal 33Rsun is necessary for a quantitative comparison. The period of pulsation changes very little for different l values and therefore the period-luminosity relation that is determined from radial oscillations can be used for nonradial oscillations as well. If we use a period-radius relation for population I Cepheids we will get a radius of about 21Rsun for the fundamental period 1.49days. From the same relation we must have a fundamental mode period 3.2days if the radius is 33Rsun. Burki,Mayor and Benz(1982) have also determined the radius of HR7308 and they suggest that the star is pulsating in the second harmonic radial mode because they have also a similar problem with their value of the radius 32Rsun. If we admit us to use the relation between the fundamental and second harmonic mode periods from Table 28 we will get a fundamental mode period equal 2.71days for the radial and 2.89days for the nonradial oscillations with l=4 if the second harmonic period is 1.49days. The corresponding values of the radius is 30Rsun and 31Rsun respectively in good agreement with the determinations. If we can extrapolate the results from the models by Osaki we find that only the first harmonic mode is pulsationally unstable but the oscillations in the fundamental mode will be nearly 4 times faster damped than in the case of the second harmonic mode for l=4. The fundamental period corresponding to the first harmonic period 1.49days is from Table 28 equal to 2.02days which gives the value of the radius equal 25Rsun. It is necessary to compute realistic models for a Cepheid with 33Rsun and oscillating in the second harmonic mode with l=4 to clear up this problem. The existence of companions that disturbe the pulsations makes the situation more complex and may explain the failure of ordinary Cepheid models in the case of HR7308.

ESTIMATION OF THE MASS

The mass of the Cepheid in HR7308 will be computed from evolutionary models and from the pulsational theory for different modes of pulsation.

The classical statement of the dependence of the pulsation period of a star on its other properties is Eddington's result,

$$P = Q * \left(\frac{rho}{rhosun}\right)^{-\frac{1}{2}} \qquad (35)$$

where Q is a constant. This is not generally valid for all stars in the Cepheid instability strip and Cogan(1970) has investigated the dependence of the pulsation "constant" Q,upon mass,luminosity and radius by solving the linear adiabatic pulsation equation for 53 model envelopes. He writes equation (35) as

$$\log P - \log R = \log Q - 0.5 * \log M/R \qquad (36)$$

where the left-hand side is determined by the observations and the right-hand side depends only on M/R. However,Faulkner(1977b) has determined fitting formulas for Q that represent the model computations by Cox,King and Stellingwerf(1972) and King,Hansen,Ross and Cox(1975) better than 0.5%. The models have the compositions (Y=0.354,Z=0.044),(Y=0.28,Z=0.02) and (Y=0.18,Z=0.02) and Q values for the fundamental,first harmonic and second harmonic can be computed. I have calculated interpolation formulas that reduce the 63 constants given by Faulkner to 21 formulas with 63 constants. These formulas and formula (6)-(10) by Faulkner are used to compute values of Q for given values of P,M,R,log Te,log L,Y,Z and pulsational mode. It is found that the fundamental mode is very insensitive to composition and a good first approximation for the mass can be computed. From the observed period ratios P1/P0 and P2/P0 tabulated by Faulkner(1977a) one can compute the mass for all three periods. Using the values of P,R,log Te,log L,Y and Z derived below from evolutionary theory,it is found that the abundance of Y or Z or both must be increased in the pulsating envelope compared to the original composition of the matter from which the star was formed. I have prefered to keep Z fixed and increase only the abundance of Y. Y and M are optimized in an iterative way and a typical value of Y in the pulsating envelope is 0.50 for Z=0.02. The determined Y values may in fact not exactly represent the true composition in the pulsating envelope of the Cepheid because it is linearly extrapolated. Inhomogenous models of Cepheid envelopes with an increased abundance of helium at some levels are computed by Cox,Michaud and Hodson(1978); they can reproduce the observed period ratios and remove the discrepancy between pulsational and evolutionary masses. Stothers(1979) on the other hand has shown that the problem with the massdetermination from period ratios can also be solved by a homogenous model if a reasonably strong axisymmetric magnetic field exist in the envelope. The present computations support the assumption of helium enrichment in the envelope although other explainations for the mass discrepancy cannot be ruled out completely.

The evolutionary masse has been determined from the models published by Becker(1981), computed by Becker,Iben and Tuggle(1977). Becker has tabulated time after ZAMS,logTe,log(L/Lsun),R/Rsun,log g and parameters describing the internal structure for different masses and compositions. The fundamental period is Pf in the formulas below. Iben and Tuggle(1975) have given a formula for log Pf as a function of dlog L and dlog Te:

$$\log Pf=0.713+(-3.34-1.10\,dlogL-0.16(dlogL)^2)\,dlogTe+(-0.62-0.26\,dlogL+1.9\,dlogTe)\,dlogM+(0.85+0.07\,dlogL)\,dlogL \qquad (37)$$

(dlogTe=logTe-3.75, dlogL=logL-3.25, dlogM=logM-0.70)

I have calculated the time derivative of (37) to get dP/dt. In the resulting formula (38) dM/dt has been put equal to zero, because no certain evidence exists for a substantial mass-loss during the Cepheid phase of the stellar evolution.

$$dP/dt=2.3026((-1.1-0.32\,dlogL)(d(logL)/dt)\,dlogTe+(-3.34-1.10*dlogL-0.16(dlogL)^2)(d(logTe)/dt)+(-0.26(d(logL)/dt)+1.9(d(logTe)/dt))*dlogM+(0.87+0.14\,dlogL)(d(logL)/dt)) \qquad (38)$$

From these formulas and the tables by Becker it is possible to compute the secular variation of the pulsational period. The secular variation of the period is determined to be 1.2exp(-6)day/year from the observations and the period is increasing. This tells us that HR 7308 is on its first,third or fifth crossing of the instability strip. A computation of the theoretical value of the period and its secular variation can be useful for the identification of the actual crossing. The secular variation gives also an independent way to determine the luminosity without any a priori knowledge of the pulsational mode. The secular variation and log Te give a value of the luminosity for every composition given by Becker.

Even if the tables by Becker are quite extensive it is not a trivial problem to make the necessary interpolations. At some parts along the tracks the parameters change rapidly and it is therefore necessary to make a lot of "local" solutions. Least square fits to a line or a parabola of dP/dt and P are computed from three or four tabulated values of log L and log Te. The fitting formula for (dP(t)/dt)/P(t) is solved for the time t after ZAMS that gives the observed value of the relative secular variation. The values of log L and log Te at that time are computed from similar least square fits to these quantities.

This computation is made for different masses but the same Y and Z and a line is drawn in the log L - log Te diagram through the points with the observed (dP/dt)/P values. The point where this line crosses the line defined by a certain log Te value gives the value of log L for a Cepheid with that composition and effective temperature. The mass is interpolated from the masses of the two closest evolutionary tracks in a logarithmic scale. In general two solutions exist for every composition one at the

blue end and the other at the red end of the loops. The solution at the blue end is always well-defined but the solution at the red end is sometimes useless because the line that defines the solution is almost vertical and parallell to the luminosity axis. If the solution falls outside the instability strip it is of course useless and it is also possible to exclude solutions that give values of log L differing too much from the P-L relation. In Table 29 the solutions that correspond to the observed value of (dP/dt)/P = 0.80exp(-6)/year are given.

Table 29

Y	Z	log L	M	Crossing	log Te
0.200	0.001	0.757	0.67	3	3.780
0.200	0.020	3.087	5.81	3	3.780
0.280	0.001	2.635	2.36	3	3.780
0.280	0.010	3.161	4.11	3	3.780
0.280	0.020	3.589	6.53	3	3.780
0.280	0.030	3.515	7.57	3	3.780
0.360	0.020	3.465	5.18	3	3.780
0.200	0.001	0.843	0.72	3	3.790
0.200	0.020	3.113	5.88	3	3.790
0.280	0.001	2.663	2.41	3	3.790
0.280	0.010	3.190	4.19	3	3.790
0.280	0.020	3.606	6.59	3	3.790
0.280	0.030	3.537	7.64	3	3.790
0.360	0.020	3.486	5.25	3	3.790
0.200	0.001	2.060	1.83	5	3.780
0.280	0.001	2.827	2.73	5	3.780
0.280	0.010	2.805	3.10	5	3.780
0.280	0.020	3.491	6.03	5	3.780
0.280	0.030	3.879	9.11	5	3.780
0.360	0.020	3.465	5.18	5	3.780
0.200	0.001	2.110	1.90	5	3.790
0.280	0.001	2.850	2.78	5	3.790
0.280	0.010	2.850	3.20	5	3.790
0.280	0.020	3.511	6.11	5	3.790
0.280	0.030	3.894	9.17	5	3.790
0.360	0.020	3.531	5.34	5	3.790

For every crossing and log Te a multiple linear regression is made that gives log L and M as a function of Y,Z and log Te. The third crossing and $(dP/dt)/P=0.80\exp(-6)$/year gives:

$$\log L=-6.553+43.71Y+338.5Z-41.48Y^2-1715.9Z^2-906.9YZ+3.276\log Te \quad (39)$$

$$\log M=-10.77+78.87Y+473.1Z-121.3Y^2-2534.1Z^2-744.1YZ+6.367\log Te \quad (40)$$

The fifth crossing and $(dP/dt)/P=0.80\exp(-6)$/year gives:

$$\log L=-2.259+29.81Y+10.81Z-43.89Y^2+928.3Z^2+3.628\log Te \quad (41)$$

$$\log M=-5.69+55.21Y+26.59Z-95.32Y^2+6510.1Z^2+8.550\log Te \quad (42)$$

The mass-luminosity relation can be expressed as:

$$d\log M=\alpha+\beta(d\log L) \quad (43)$$

Iben and Tuggle(1975) give the theoretical PLTe relation:

$$\log Pf=0.713-0.62\alpha+(-3.34+1.9\alpha+(-1.10+1.9\beta)d\log L)d\log Te$$

$$+(0.85-0.26\alpha-0.62\beta)\,d\log L \qquad (44)$$

Becker,Iben and Tuggle(1977) give the following relations for alpha and beta:

$$\alpha=0.0659+7.83dZ(1+36.7dZ)-0.873dY(1+7.7dY) \qquad (45)$$

$$\beta=0.219-4.08dZ(1+34.9dZ)+0.053dY(1+66.8dY) \qquad (46)$$

$(dY=Y-0.28\ ,\ dZ=Z-0.02)$

From (44) log L can be expressed as a function of Pf,Te,Y and Z as:

$$d\log L = \frac{(\log Pf-(0.713-0.62\alpha+(-3.34+1.9\alpha)\,d\log Te))}{((-1.10+1.9\beta)\,d\log Te+(0.85-0.26\alpha-0.62\beta))} \qquad (47)$$

The surface-brightness technique for Cepheids gives log L from the observations of V,B-V and radial velocity for different values of E(B-V) and Z. Equation (48) is the result of a least-square fit:

$$d\log L = -0.770 + 4.011 d\log Te \qquad (48)$$

The log Te values are computed from the well known calibration by Kraft(1961) and with a correction for metal content from Iben and Tuggle(1975) given as:

$$\log Te = 3.886 + 1.33(Z-0.02) - 0.175\langle B-V\rangle_0 \qquad (49)$$

Sandage and Tammann(1969) give the bolometric correction:

$$B.C.=0.145 - 0.322(B-V)_0 \qquad (50)$$

$$\log(L/Lsun)= 0.4(4.77-Mbol) \qquad (51)$$

The equations (39) for the third and (41) for the fifth crossing combined with (47)-(51) offer a possibility to derive a unique solution for log L ,Y and Z for given values of Pf and $(B-V)_0$. The equations (39),(41) and (48) have numerical constants valid only for HR7303 and they must be determined for every specific Cepheid. The result of the computations with different values of Pf and E(B-V) is presented in Table 30.

Table 30

Pf(d)	E(B-V)	Log Te	Y	Z	Log L	M	Crossing	Remark
2.89	0.02	3.7807	0.2018	0.0202	3.103	5.7	3	
2.89	0.07	3.7881	0.2121	0.0191	3.136	5.6	3	
2.89	0.12	3.7951	0.2229	0.0179	3.160	5.5	3	
2.89	0.17	3.8061	0.2292	0.0172	3.192	5.5	3	FBE
2.79	0.07	-	-	-	-	-	3	Y<0.20
2.89	0.07	3.7881	0.2121	0.0191	3.136	5.6	3	
2.99	0.07	3.7833	0.2397	0.0156	3.111	5.0	3	
3.09	0.07	3.7810	0.2512	0.0138	3.108	4.8	3	
2.89	0.02	3.7809	0.2414	0.0204	3.104	5.3	5	
2.89	0.07	3.7888	0.2466	0.0197	3.136	5.3	5	
2.89	0.12	3.7964	0.2527	0.0188	3.165	5.3	5	
2.89	0.17	3.8041	0.2592	0.0180	3.195	5.3	5	FBE
2.49	0.07	-	-	-	-	-	5	Y<0.20
2.59	0.07	3.7991	0.2042	0.0275	3.177	6.5	5	FBE
2.69	0.07	3.7957	0.2181	0.0249	3.164	6.1	5	
2.79	0.07	3.7924	0.2319	0.0223	3.151	5.7	5	
2.89	0.07	3.7888	0.2466	0.0197	3.136	5.3	5	
2.94	0.07	3.7874	0.2525	0.0187	3.130	5.2	5	Best
2.99	0.07	3.7853	0.2618	0.0171	3.121	4.9	5	
3.09	0.07	3.7826	0.2750	0.0150	3.111	4.6	5	

All the data in Table 30 give the same radius 32.5Rsun as computed from the Stephan-Boltzmann law. The determinations of the mass for HR7308 by both the pulsational and evolutionary methods indicates that the Cepheid is pulsating in the second harmonic mode and that the radius is 32.5Rsun which can be compared to the determination by the Wesselink method at Pa-phase 0.70 by Burki,Mayor and Benz(1982).

This principle has also been tested for Delta Cep and Zeta Gem. The solutions show that both stars are pulsating in the fundamental mode and are on the second crossing of the instability strip. A comparison between the results from the different stars can be made in Table 31.

Table 31

Property	HR7308	Delta Cep	Zeta Gem
Fundamental period	2.94d	5.336d	10.15d
Radius(optimized)	32.3Rsun	46.8Rsun	69.5Rsun
Mass	5.2Msun	5.3Msun	5.7Msun
log(L/Lsun)	3.13	3.35	3.51
Y(initial)	0.253	0.372	0.335
Z(initial)	0.0187	0.0240	0.0170
Y(envelope)	0.51	0.45	0.51
Crossing of instability strip	5	2	2

It seems from Table 31 as if HR7308 have a significantly lower abundance of Helium compared to the other two high amplitude stable Cepheids and that may be one possible explaination for the extraordinary amplitude variation of HR7308.

An advantage with the above described technique is that it is not necessary to know in which mode the Cepheid is pulsating and it may be possible to identify the mode,which will reduce the the uncertainty in the period-luminisity relation especially for short periods.

Before HR7308 was know as Cepheid the luminosity was determined by Baker(1974) and Sorvari(1974) from the luminosity sensitive OI-triplett at 7774Å. The most reasonable calibration is that of Sorvari and he has determined MV=-2.96 which corresponds to logL/logLsun=3.07. The epoch of the observation is not published. However, it is not clear how accurate this calibration is but the agreement with the above result is very good.

CONCLUSIONS

The studies of HR7308 has revealed a most remarkable star; a 'classical' Cepheid with the shortest known period,pulsating nonradially in the second harmonic mode,on its fifth crossing of the instability strip and with variable amplitude. The extensive photometric and radial velocity data at hand allow the determination of most physical properties of the star, including the radius,mass,luminosity,effective temperature, initial chemical composition as well as several periodical variations with long periods possibly related to one or two companions with smaller mass and luminosity than the Cepheid. The determined properties of the Cepheid are summarized in Table 32.

Table 32

Observed period (second harmonic)	1.490795+1.22*exp(-6)(J.D.-2443600/365.25day
Computed fundamental period	2.94days
Computed first harmonic period	2.08days
Period of amplitude variation	1178.5days=3.2266years
Periods of period variation	1178.5days and 2655days
Secular variation of the period	+1.22*exp(-6)d/year=0.105s/year
Spherical harmonic orders at maximum amplitude	l=2 m=0;oscillating along the axis of rotations
Spherical harmonic order at minimum amplitude	l=4 m=2;oscillating along the axis of rotation and along the equator
Inclination of the axis of rotation	71degrees
Absolut V magnitude	-3.04
Bolometric magnitude	-3.07
log L(L/Lsun)	3.14
Distance	623pc
Radius	33Rsun
Mass	5.2Msun
Mean effective temperature(Kraft)	6155K
Mean effective temperature(optimized in the evolutionary calculations)	6129K
log Q0	-1.435
log Q1	-1.585
log Q2	-1.730
Initial composition:	
X=Hydrogen	0.728
Y=Helium	0.253
Z=metals	0.0187
Evolutionary state	5 crossing of instability strip

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